Fiscal Expenditure Consolidation and Sovereign Debt Restructurings: Ex Ante or Ex Post*

Preliminary and Incomplete

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Abstract

Sovereigns implement fiscal expenditure consolidation before or upon debt crises. We compile data on fiscal expenditure consolidation around sovereign debt restructurings with private external creditors in 1975–2020. We find that (i) expenditure consolidation precedes preemptive restructurings—"*ex ante*"—, while occurs upon defaults/post-default restructurings—"*ex post*"—; and (ii) public investment dynamics and restructuring duration differ between preemptive and post-default restructurings. We build sovereign debt model with endogenous choice of preemptive and post-default renegotiations and public capital accumulation. Our model quantitatively shows that high public capital allows the sovereign to implement ex ante expenditure consolidation which succeed in subsequent preemptive restructurings. Data support theoretical predictions.

JEL Classification Codes: F34, F41, H63

Key words: Fiscal Austerity; Expenditure Consolidation; Sovereign Debt; Sovereign Default; Public Investment;

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1 Introduction

Sovereigns implement fiscal expenditure consolidation prior to or upon debt crises. We compile data on fiscal expenditure consolidation around sovereign debt restructurings with private external creditors in 1975–2020. We find that (i) expenditure consolidation precedes preemptive restructurings—"*ex ante*"—, while occurs upon defaults/post-default restructurings—"*ex post*"—; and (ii) public investment dynamics and restructuring duration differ between preemptive and post-default restructurings. We build a sovereign debt model that embeds explicitly endogenous choice of preemptive and post-default renegotiations and public capital accumulation. Our model quantitatively shows that high public capital allows the sovereign to implement ex ante expenditure consolidation which succeed in subsequent preemptive restructurings. Data support our theoretical predictions.

We first present two new datasets on (i) non-restructuring debt distress episodes and (ii) fiscal expenditure consolidation around sovereign debt restructurings for 75 countries experiencing at least one external debt restructuring with private creditors in 1975–2020. On the former, we define non-restructuring debt distress satisfying three criteria consistent with a debt restructuring and compile 25 episodes. On the latter, we apply fiscal consolidation criteria in Alesina and Perotti (1997) specifically on public expenditure and compile fiscal expenditure consolidation episodes.[1] We merge fiscal expenditure consolidation episodes with three types of sovereign debt crises, i.e., post-default restructuring, preemptive restructuring and non-restructuring debt distress in Asonuma and Trebesch (2016) and our new dataset. We classify eight joint strategies of fiscal expenditure consolidation and debt restructurings differentiated by timing of fiscal expenditure consolidation and debt restructuring strategies.

Our comprehensive datasets show five new stylized facts. First, there are three main strategies: (a) ex post fiscal expenditure consolidation and post-default restructuring, (b) ex ante fiscal expenditure consolidation and preemptive restructuring, and (c) ex ante fiscal expenditure consolidation and no restructuring (and debt distress). Second, public investment declines sharply prior to the start of preemptive restructurings, while upon the start of post-default restructurings. Third, debt settlement is quick and prior to recoveries in public investment in preemptive cases, while protracted and after recoveries in public investment in post-default cases. Fourth, recoveries in public investment are quicker in preemptive cases than in post-default cases. Fifth, public consumption and transfers decline temporarily and recover quickly in both types of restructurings.

These stylized facts uncover a new aspect of fiscal austerity and sovereign debt, which the literature has not explored fully yet. These facts pose two questions: Why ex ante fiscal consolidation precedes a preemptive restructuring, while ex post fiscal consolidation occurs upon default/post-default restructuring? If so, why is not always fiscal expenditure consolidation ex ante? While we consider these questions, a more fundamental question in the literature emerges: What explains the sovereign's joint choice of fiscal expenditure consolidation and debt

[1] Ongoing work Asonuma, Joo and Zhang (2022) compile a new dataset on fiscal primary balance, revenues, and expenditure and code revenues-based and expenditure-based consolidation.

restructuring? These questions pose a challenge to the conventional views in literature that fiscal expenditure consolidation often takes place after the sovereign's default choice in emerging market countries (EMs). To our knowledge, our paper is the first to explore the sovereign's joint decision of fiscal expenditure consolidation and debt restructuring.

To answer these questions, we build a theoretical sovereign debt model that embeds endogenous public capital accumulation, expenditure composition, production and choice of preemptive and post-default renegotiations. In line with recent quantitative work on sovereign debt, our model follows a traditional framework of Eaton and Gersovitz (1981). We follow closely two modeling approaches in the literature: (i) one with a critical role for fiscal policy—in a setup with distinct private and public sectors, distortional tax and two types of consumption (Cuadra et al. 2010; Arellano and Bai 2017) and (ii) one with multi-round debt renegotiations (Benjamin and Wright 2013).

Our theoretical innovation is to newly introduce an interaction between the sovereign's endogenous choice of preemptive and post-default restructurings—made at different points in time (i.e., prior to and after productivity realization)—and endogenous public capital accumulation. In this regard, our model is different from (i) Asonuma and Trebesch (2016) with endogenous choice of preemptive and post-default restructurings and exogenous income and (ii) Gordon and Guerron-Quintana (2018) with endogenous choice of default and private capital accumulation. The sovereign's choice of two types of restructurings and public capital accumulation mutually interact to newly account for (b) ex ante fiscal expenditure consolidation and preemptive restructuring, and (c) ex ante fiscal expenditure consolidation and no restructuring (debt distress) as observed in data. This differentiates our model from previous studies on sovereign debt and fiscal policy (e.g., Cuadra et al. 2010) which account for only (a) ex post fiscal expenditure consolidation and post-default restructuring.

Our model presents two main theoretical predictions. First, our model shows predictions on the sovereign's choice of fiscal expenditure consolidation and debt restructurings, especially (b) ex ante fiscal expenditure consolidation and preemptive restructuring and (c) ex ante fiscal expenditure consolidation and no restructuring (debt distress). When debt is high, public capital is high and productivity is moderate/high, the sovereign repays debt in full, issues less new debt and implements ex ante fiscal expenditure consolidation. Under high public capital (i.e., low marginal product of capital), benefits of reducing debt burden and default risk by reducing public investment (i.e., ex ante expenditure consolidation) are higher than those of maintaining public investment unchanged (i.e., no ex ante expenditure consolidation). In a subsequent period, before productivity materializes, when debt remains high, the sovereign opts a preemptive restructuring—"ex ante fiscal consolidation and preemptive restructuring". After productivity materializes as moderate/high, when debt is moderate, the sovereign repays debt in full—"ex ante fiscal consolidation and no restructuring (debt distress)".

Our model also shows conventional (a) ex post fiscal expenditure consolidation and default/post-default restructuring. When debt is moderate and/or public capital is low, and productivity is moderate/high, the sovereign does not implement ex ante fiscal expenditure consolidation.

Under low public capital (i.e., high marginal product of capital), benefits of maintaining public investment unchanged are higher than those of reducing debt burden and default risk by reducing public investment. In a subsequent period, after productivity materializes as low, when debt is moderate, the sovereign opts to default and has no option but to reduce public investment—"ex post fiscal consolidation and default/post-default restructuring".

Second, our model makes predictions on the role of public capital (investment dynamics) determining the timing of fiscal expenditure consolidation and associated debt crisis resolution. High public capital allows the sovereign to implement ex ante fiscal expenditure consolidation through reduction in public investment ("fiscal consolidation channel"). After ex ante fiscal expenditure consolidation, it results in low level of public capital. It is more willing to take a preemptive restructuring in subsequent periods because it opts to avoid larger costs associated with a default/post-default restructuring—longer periods of financial autarky with larger productivity losses—when public capital is low. Under preemptive restructuring, it achieves both quick debt settlement and public investment recovery due to lower productivity losses and no need for further fiscal expenditure consolidation.

On the contrary, when public capital is low, the sovereign refrains ex ante fiscal expenditure consolidation. After productivity materializes as low and it opts to default, it has no choice other than reduce public investment given loss in market access and larger productivity losses. Under post-default restructuring, it takes time to recover public investment and achieve debt settlement due to larger productivity losses and even lower level of public capital due to ex post fiscal expenditure consolidation.

The sovereign benefits from high public capital through the fiscal consolidation channel by choosing desirable timing of fiscal expenditure consolidation (i.e., ex ante or ex post). This differs from three conventional channels of public capital. The sovereign benefits from high capital by improving its capacity of repayment ("smoothing channel"). Alternatively, it also benefits from high public capital through smoothing household consumption in financial autarky ("autarky channel") and quick debt settlement ("renegotiation channel").

Our theoretical predictions are supported by data: both the sovereign's choice on ex ante and ex post expenditure consolidation and the consequences of these two types of consolidation. The quantitative analysis calibrated to the Argentine post-default restructuring in 2001–05 and the Uruguayan preemptive restructuring in 2003 replicates the five stylized facts: (i) three dominant strategies of expenditure consolidation and debt restructuring, (ii) a sharp decline in public investment prior to a preemptive restructuring and a recovery in public investment after quick debt settlement, (iii) a sharp decline in public investment upon a post-default restructuring and a gradual recovery in public investment leading to debt settlement; (iv) a quicker recovery in public investment in preemptive restructurings than in post-default episodes, and (v) a short-lived decline and a quick recovery in consumption and transfers in both types of restructurings.

Literature Review First, current paper contributes to empirical literature on fiscal austerity. Using an enrich dataset of austerity (consolidation) plans in advanced economies. Gua-

jardo et al. (2014) find that fiscal consolidation has contractionary effects on private demand and GDP. Alesina et al. (2019) show larger negative effects of tax-based austerity on GDP growth than those of expenditure-based austerity. Our paper fills a gap in the literature by showing new empirical facts on different consequences of two types of fiscal expenditure consolidation (i.e., ex ante vs. ex post) on sovereign debt crises and resolution.

Second, a theoretical strand of literatures explores the sovereign's interdependent choice of debt (foreign borrowing), defaults, and fiscal policy. Recent studies, e.g., Cuadra et al. (2010), Arellano and Bai (2017), Bianchi et al. (2020) and Hatchondo et al. (forthcoming) embed fiscal policy instrument (i.e., expenditure and taxation) in a model wtih endogenous default and production.[2] Our paper contributes to the literature in that we newly explain fiscal expenditure consolidation prior to debt restructurings, i.e., ex ante expenditure consolidation.

Lastly, the theoretical work on sovereign debt restructuring explains different types of defaults in the context of a bargaining between a sovereign debtor and its creditors. Arellano et al. (2019) explain both "partial default" and "full default" in that these are differentiated by debt repayment amount, (i.e., partial or no) upon the sovereign's choice of default. Hatchondo et al. (2014) show both "voluntary debt exchange" and default that both occur at the same time upon income realization, i.e., ex post. On the contrary, Asonuma and Trebesch (2016) explain both preemptive and post-default debt restructurings that occur at different point of times, prior to and after income realization. We newly find how the sovereign's restructuring choice is jointly determined with expenditure consolidation choice and both have consequences to sovereign debt crises and resolution.

2 Dataset and Stylized Facts

2.1 New Dataset on Non-Restructuring Debt Distress Episodes

Table 1: Non-restructuring Debt Distress Episodes in 1975–2020

	Observation	Country	Duration (years, average)	EMBIG Bond Spreads (basis points, average)	Estimated Probability[1/] (percent, average)
Non-restructuring Debt Distress	25	19	1.4		
EMBIG bond spreads	8	6	1.5	1287	
Estimated restructuring probability	18	15	1.3		21.1/38.8
Debt Restructuring	197	75	3.4	1098	14.2/36.4

Sources: Asonuma and Trebesch (2016), our calculation
Note: [1/] Estimated probability for post-default (left) and preemptive restructurings (right).

First, we compile a new dataset on non-restructuring debt distress episodes in 1975–2020. To have these episodes as a complement to restructurings, we classify these episodes based on three criteria: (i) high expected restructuring probability—prior to the event, sovereigns anticipate that a restructuring is highly likely as in actual restructuring events—; (ii) no overlap with restructurings—duration of debt distress and debt restructuring does not overlap—; (iii)

[2]See also Kaminsky et al. (2005) and Goncalves and Guimaraes (2015).

being cured—there is an interval of at least 2 years (i,e., non-distressed period) prior to a new restructuring or debt distress.

For criterion (i), we use bond spreads from the J.P. Morgan Emerging Market Bond Index Global (EMBIG) in 1993–2020 for 40 countries with debt restructurings. Average bond spreads over 12 months prior to the start of debt restructurings in the same sample period are 1098 basis points. We set this as a threshold for annual bond spreads. To complement both limited coverage of time and countries, we follow Asonuma et al. (2021) to apply a probit regression for preemptive and post-default restructurings for 75 countries in 1975–2020. Average estimated probability of these restructurings is 36.4 and 14.2 and we set this as a threshold for expected restructuring probability. For criteria (ii) and (iii), we use restructuring duration dataset in Asonuma and Trebesch (2016).

We find 25 non-restructuring debt distress episodes in 19 countries. Duration of debt distress is 1.4 years on average. Table A1 in Appendix A.1 report all debt distress events and the underlying criteria met.

2.2 New Dataset on Fiscal Expenditure Consolidation

Second, we code a new dataset on fiscal expenditure consolidation around debt restructurings, non-restructuring debt distress and non-debt crisis recessions in 1975–2020 for 75 countries. Following classification of a tight fiscal policy in Alesina and Perotti (1997), we define fiscal expenditure consolidation:

- ***Definition 1***: Fiscal expenditure consolidation is a year in which a *cyclical adjusted* fiscal expenditure-to-GDP ratio falls by more than 1.5 percent or two sequential years in which a *cyclical adjusted* fiscal expenditure-to-GDP ratio falls by at least 1.25 percent a year.

When a sovereign government initiates a debt restructuring, it often suffers large external shocks as a result, a sharp GDP contraction. To minimize the effects of the GDP contraction on the fiscal expenditure-to-GDP ratio, we complement the conventional definition of fiscal consolidation with an alternative classification based on fiscal expenditure-to-lagged GDP ratio.

- ***Definition 1'***: Fiscal expenditure consolidation is a year in which a fiscal expenditure-to-*lagged* GDP ratio falls by more than 1.5 percent or two sequential years in which a fiscal expenditure-to-*lagged* GDP ratio falls by at least 1.25 percent a year.

Our fiscal expenditure consolidation is based on actual public expenditure series which includes a sovereign government's fiscal policy response to business cycles (i.e., "endogenous" response). This is different from fiscal consolidation in the literature of fiscal multiplier which is based on the sovereign government's planned fiscal consolidation independent from business cycles (e.g., Alesina et al. 2019).

We compile 148, 14, and 194 fiscal expenditure consolidation episodes around debt restructurings, non-restructuring debt distress and non-debt crisis recessions—from 3 years before each

event to the end of event duration. Fiscal expenditure consolidation is more frequent (75 percent of events) than non-restructuring debt distress and non-debt crisis recessions (57 or 60 percent). Table A2 in Appendix A.2 report examples of fiscal expenditure consolidation episodes

Table 2: : Fiscal Expenditure Consolidation Episodes around Debt Restructurings, Debt Distress and Recessions in 1975–2020

	Event	Fiscal Expenditure Consolidation	
	Observation	Observation	Share (% of event)
Debt Restructuring	197	148	75.1
Non-restructuring Debt Distress	25	14	57.0
Non-debt Crisis Recession	325	194	59.7

We distinguish two types of fiscal expenditure consolidation by timing:

- ***Definition 2***: Ex ante fiscal expenditure consolidation is one which occurs prior to the start of a debt crisis (i.e., either a start of debt restructuring or a default). Ex post fiscal expenditure consolidation is one which occurs up or after the start of a debt crisis.

We use monthly sovereign default and debt restructuring database in Asonuma and Trebesch (2016) and our non-restructuring debt distress episodes, and compile two types of fiscal expenditure consolidation. In the case of non-restructuring debt distress, all fiscal expenditure consolidation around the event is defined as ex ante because no debt crisis occurs. We merge fiscal expenditure consolidation episodes with the dataset on restructuring strategies in Asonuma and Trebesch (2016). Asonuma and Trebesch (2016) define a preemptive restructuring as a restructuring without missed payments, and a post-default restructuring as a restructuring that a sovereign unilaterally defaults on its payments. Our consolidated datasets provide eight joint strategies of fiscal expenditure consolidation and debt restructurings distinguished by timing and restructuring strategies in Appendix A.2.

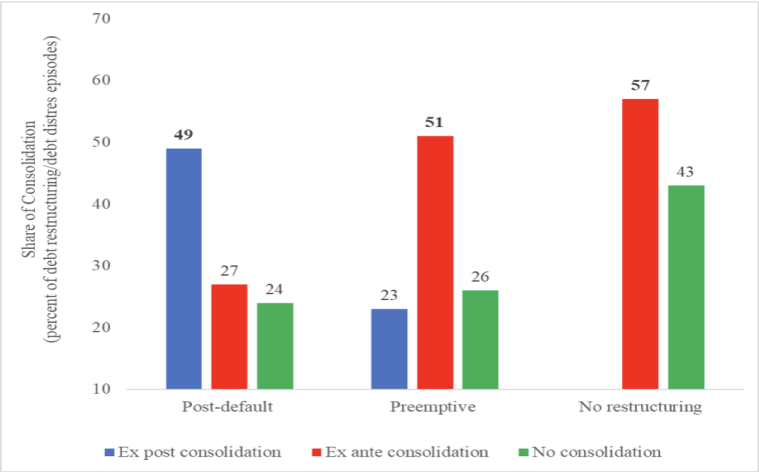
2.3 Empirical Findings: Five Stylized Facts

Our findings for fiscal expenditure consolidation and debt restructurings in 1975–2020 can be summarized in five main stylized facts.

- ***Stylized fact 1: There are three main strategies of fiscal expenditure consolidation and sovereign debt restructurings.***

Figure 1 reports share of fiscal expenditure consolidation choice for each restructuring choice—adds up to 100 percent for post-default, preemptive and no restructuring choice. Ex post expenditure consolidation is the most frequent accounting for 49 percent of 111 post-default restructurings (left section). On the contrary, ex ante expenditure consolidation is the most frequent accounting for 51 percent of 75 preemptive restructurings and 57 percent of 25 non-restructuring debt distress episodes (center and right sections). Figure B1 in Appendix B confirms that the

Figure 1: Strategies of Fiscal Expenditure Consolidation and Sovereign Debt Restructurings in 1975–2020



observed pattern remains the same when we apply an alternative classification of expenditure consolidation using expenditure-to-lagged GDP ratio. Figure B2 in Appendix B reports strategies of fiscal consolidation around non-debt crisis recessions.

- ***Stylized fact 2: Public investment declines sharply ex ante in preemptive cases, while ex post in post-default cases.***

- ***Stylized fact 3: Debt settlement takes place before recoveries in public investment in preemptive cases, while after in post-default cases.***

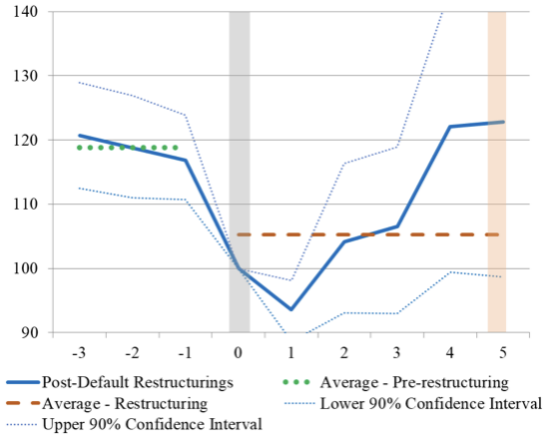
Figure 2 shows the dynamics of public investment around preemptive, post-default restructurings, and non-restructuring debt distress. In three panels, the start and end of the debt restructurings and debt distress are marked by gray and orange vertical bars, respectively. Public investment is in real and level terms and is normalized at levels at the start of debt restructurings and debt distress. The red, blue and purple solid lines show an average for all preemptive, post-default restructuring and non-restructuring debt distress episodes for which public investment data are available. The green dotted and brown dashed lines show average public investment during the pre-restructuring (pre-debt distress) and restructuring (debt distress) periods.

Public investment declines markedly prior to preemptive restructurings i.e., from year -3 to year -1 (panel ii), while at the onset of post-default restructuring i.e., from year -1 to year 1 (panel i). On the contrary, public investment reduces temporarily and marginally upon non-restructuring debt distress (panel iii).

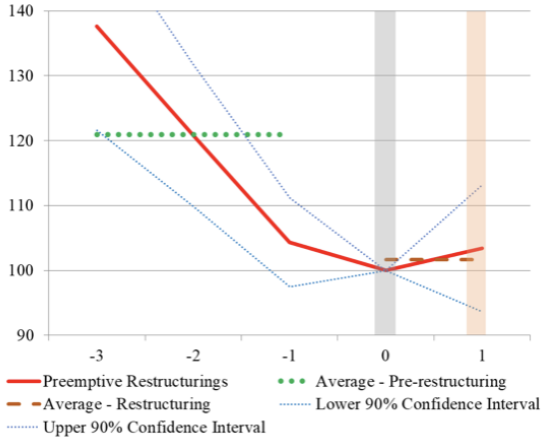
In preemptive restructurings, public investment recovers only partially in year 1 and debt settlement takes place in year 1 leading to full recoveries in public investment afterwards (panel ii). In post-default restructurings, public investment recovers to the pre-crisis level in year 4 and debt settlement follows in year 5 (panel i). On the contrary, in non-restructuring debt distress,

Figure 2: Public Investment

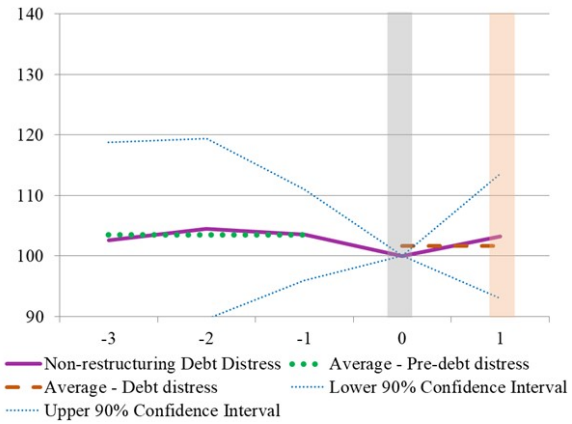
(i) Post-default Restructurings



(ii) Preemptive Restructurings



(iii) Non-restructuring Debt Distress



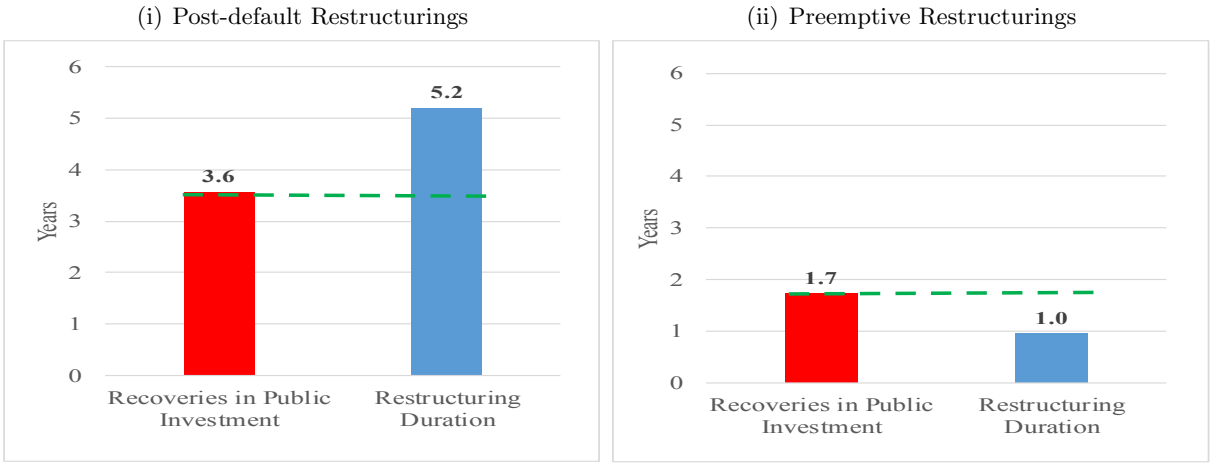
public investment recovers to pre-crisis level and non-restructuring debt distress ends in year 1. When we measure public investment as percent of GDP, we also find the identical dynamics of public investment-to-GDP ratio for two types of debt restructurings and non-restructuring debt distress (Figure B3 in Appendix B). Figure B4 in Appendix B reports public investment dynamics around non-debt crisis recessions.

- ***Stylized fact 4: Recoveries in public investment are shorter in preemptive cases than in post-default cases.***

Figure 3 contrasts length of recoveries in public investment between two types of restructurings. Recoveries in public investment are shorter in preemptive cases than in post-default cases (1.7 years vs. 3.6 years). In preemptive restructurings, public investment recovers to the pre-crisis level immediately after debt settlement (1.7 years vs. 1.0 year in panel i) taking benefits of quick re-access to the international capital market. On the contrary, in post-default

restructurings, public investment recovers to the pre-crisis level before debt settlement (3.6 years vs. 5.2 years in panel ii).

Figure 3: Recoveries in Public Investment and Duration of Restructurings



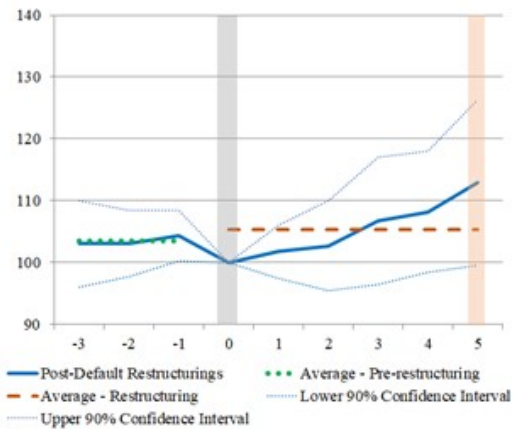
- ***Stylized fact 5: Public consumption and transfers decline temporarily ex post and recover quickly in both types of restructurings.***

Figure 4 shows the dynamics of public consumption and transfers around post-default, preemptive restructurings and non-restructuring debt distress. We follow the same presentation approach as in Figure 2 in terms of time horizon, timing of events, scale (real and level), normalization of the series at level of the start of restructurings and debt distress (=100), and average in the two periods. Public consumption and transfers fall temporarily at the onset of post-default and preemptive restructurings (year 0) and recover quickly and reach the pre-crisis level in years 1 or 2 (panels i-ii). Public consumption and transfers continue an upward trend around non-restructuring debt distress (panel iii). When we measure public consumption and transfers as percent of GDP, we also find the identical dynamics of public consumption and transfers-to-GDP ratio for two types of debt restructurings and non-restructuring debt distress (Figure B5 in Appendix B). Figure B5 in Appendix B reports dynamics of public consumption and transfers around non-debt crisis recessions.

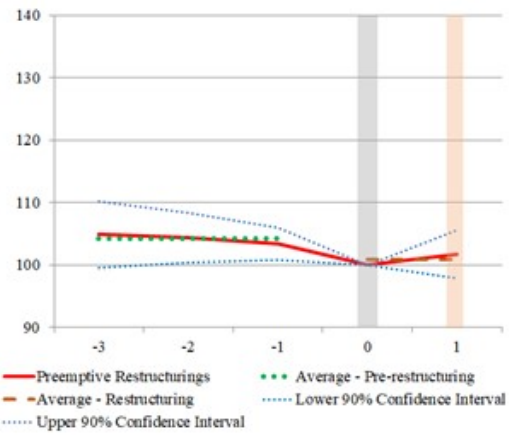
A contrast between Figures 2 and 4 shows a difference in the dynamics between public consumption and transfers, and investment. In preemptive restructurings, public investment experiences a severe decline prior to restructurings, but consumption and transfers do not. In post-default restructurings, public investment experiences a severe decline at the onset of restructurings, but consumption and transfers experience a small decline. As a result, public expenditure skews heavily towards consumption and transfers under fiscal consolidation in both preemptive and post-default cases reported in Figure B6 in Appendix B.

Figure 4: Public Consumption and Transfers

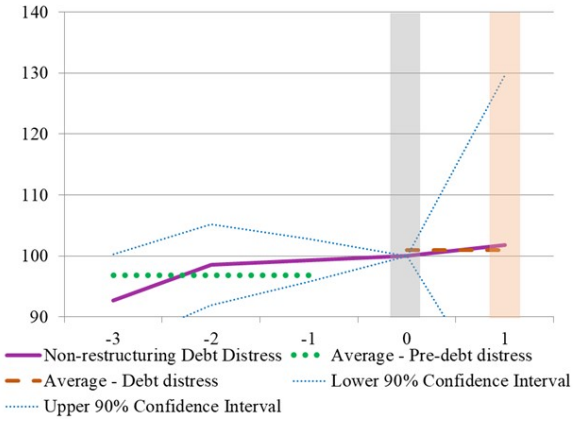
(i) Post-default Restructurings



(ii) Preemptive Restructurings



(iii) Non-restructuring Debt Distress



3 Theoretical Model

3.1 Summary of Theoretical Findings

Our model presents two main theoretical predictions. First, our model shows predictions on the sovereign's choice of fiscal expenditure consolidation and debt restructurings, especially (b) ex ante fiscal expenditure consolidation and preemptive restructuring and (c) ex ante fiscal expenditure consolidation and no restructuring (debt distress). When debt is high, public capital is high and productivity is moderate/high, the sovereign repays debt in full, issues less new debt and implements ex ante fiscal expenditure consolidation. Under high public capital (i.e., low marginal product of capital), benefits of reducing debt burden and default risk by reducing public investment (i.e., ex ante expenditure consolidation) are higher than those of maintaining public investment unchanged (i.e., no ex ante expenditure consolidation). In a subsequent period, before productivity materializes, when debt remains high, the sovereign opts a preemptive restructuring—"ex ante fiscal consolidation and preemptive restructuring". After productivity materializes as moderate/high, when debt is moderate, the sovereign repays debt in full—"ex ante fiscal consolidation and no restructuring (debt distress)".

Our model also shows conventional (a) ex post fiscal expenditure consolidation and default/post-default restructuring. When debt is moderate and/or public capital is low, and productivity is moderate/high, the sovereign does not implement ex ante fiscal expenditure consolidation. Under low public capital (i.e., high marginal product of capital), benefits of maintaining public investment unchanged are higher than those of reducing debt burden and default risk by reducing public investment. In a subsequent period, after productivity materializes as low, when debt is moderate, the sovereign opts to default and has no option but to reduce public investment—"ex post fiscal consolidation and default/post-default restructuring".

Second, our model makes predictions on the role of public capital (investment dynamics) determining the timing of fiscal expenditure consolidation and associated debt crisis resolution. High public capital allows the sovereign to implement ex ante fiscal expenditure consolidation through reduction in public investment ("fiscal consolidation channel"). After ex ante fiscal expenditure consolidation, it results in low level of public capital. It is more willing to take a preemptive restructuring in subsequent periods because it opts to avoid larger costs associated with a default/post-default restructuring—longer periods of financial autarky with larger productivity losses—when public capital is low. Under preemptive restructuring, it achieves both quick debt settlement and public investment recovery due to lower productivity losses and no need for further fiscal expenditure consolidation.

On the contrary, when public capital is low, the sovereign refrains ex ante fiscal expenditure consolidation. After productivity materializes as low and it opts to default, it has no choice other than reduce public investment given loss in market access and larger productivity losses. Under post-default restructuring, it takes time to recover public investment and achieve debt settlement due to larger productivity losses and even lower level of public capital due to ex post fiscal expenditure consolidation.

The sovereign benefits from high public capital through the fiscal consolidation channel by choosing desirable timing of fiscal expenditure consolidation (i.e., ex ante or ex post). This differs from three conventional channels of public capital. The sovereign benefits from high capital by improving its capacity of repayment ("smoothing channel"). Alternatively, it also benefits from high public capital through smoothing household consumption in financial autarky ("autarky channel") and quick debt settlement ("renegotiation channel").

3.2 Assumptions in the Model

There are four agents in the model: a household, a firm, a sovereign (government), and foreign creditors.[3] The sovereign is risk averse and cannot affect the global risk-free interest rate (r^*). Foreign creditors are risk-neutral. They can borrow or lend as much as needed at the constant risk-free interest rate in the international capital market.

In each period, a stochastic productivity shock a_t materializes. It is stochastic, drawn from a compact set $A = [a_{min}, a_{max}] \subset R$. $\mu(a_{t+1}|a_t)$ is a probability distribution of a shock a_{t+1} conditional on its previous realization a_t. In addition, the sovereign's credit record $h_t \in [0, 1, 2]$, denotes "good" when the sovereign maintains access to the international market ($h_t = 0$), "intermediate" when it has partial access due to a preemptive restructuring ($h_t = 1$), or "bad" when it loses access due to a default/post-default restructuring ($h_t = 2$). The credit record keeps track of where we are in the decision tree, in particular, timing of both decisions and debt renegotiations. The information on the country's assets, public capital, credit record, and productivity shock is symmetric and perfect for all parties.[4]

Decisions by the sovereign, household and firms are made at two points in time depending the sovereign's credit record; (i) before realization of current productivity when the credit record is intermediate, (i.e., preemptive renegotiation), or (ii) after realization of current productivity when credit record is good or bad, (i.e., repayment or post-default renegotiation).

The sovereign receives consumption tax revenues and decides expenditure composition—public consumption, investment and transfers—together with its choice of repayment and default (settlement and delay), and of external borrowing.[5] Consumption tax revenues are determined by the household's optimal choice of private consumption given a constant consumption tax rate. Public consumption and transfers are provided to the household to improve his utility directly or indirectly by smoothing private consumption, respectively. Public capital rented to the firm is accumulated through net investment and is subject to both depreciation and adjustment costs.

The household receives profits from the firm, and public consumption and transfers from the government, respectively. He chooses private consumption and labor supply, and pays consumption taxes to the government. The firm receives public capital from the government and owns

[3]In this theoretical and quantitative analysis, the term sovereign corresponds to the government.

[4]Our model does not assume any ex-ante information asymmetries among four agents or disincentives of the sovereign government that make a default unavoidable.

[5]Galli (2021) shows that multiple equilibria exist in a model with total (private) capital and foreign creditors' self-fulfilling beliefs. However, in our model, the sovereign chooses public consumption and transfers besides investment to accommodate public expenditure to net borrowing, and in turn, this results in no multiple equilibriua.

private capital which is accumulated through net investment and is subject to both depreciation and adjustment costs (Gordon and Guerron-Quintana 2018). The firm chooses labor demand and private capital, produces goods, and pays profits to the household.

The sovereign bond market is incomplete. The sovereign can borrow and lend only via one-period, zero-coupon sovereign bonds, while neither the household nor firm can.[6] b_{t+1} denotes the amount of bonds to be repaid in the next period whose set is shown by $B = [b_{min}, b_{max}] \subset R$ where $b_{min} \leq 0 \leq b_{max}$. We set the lower bound for the sovereign's bond holding at $b_{min} > -y_{max}/r^*$ which is the largest debt that the sovereign could repay. The upper bound b_{max} is the high level of assets that the sovereign may accumulate.[7] We assume $q(b_{t+1}, k^g_{t+1}, 0, a_t)$ to be price of sovereign bonds with sovereign's asset position b_{t+1}, public capital k^g_{t+1}, good credit record ($h_t = 0$), and a productivity shock a_t. The bond price is determined in equilibrium.

We assume that the creditors always commit to repay their debt. However, the sovereign is free to decide whether to repay its debt or to default. If the sovereign chooses to repay its debt, it will preserve access to the international capital market in the next period. If it chooses to default, it is then subject to both exclusion from the international capital market, direct productivity loss ($\lambda_d a_t$), and arrear accumulation.[8,9], If it chooses a preemptive restructuring (no default), it will suffer partial exclusion from the international capital market and smaller direct productivity loss ($\lambda_p a_t$), but does not accumulate arrears. The assumption of two types of productivity loss is consistent with findings in empirical literature (e.g., Asonuma and Trebesch 2016) that output losses in preemptive restructurings are lower than those in post-default restructurings.

Debt renegotiations also take place at two points in time: (i) before realization of current productivity, (i.e., preemptively), or (ii) after realization of current productivity and the sovereign chooses to default, (i.e., post-default). In both renegotiations, the sovereign and creditors negotiate a reduction of debt via multi-round bargaining. At the renegotiation, one party, who is randomly selected with exogenous and constant probability, chooses whether to propose an offer with haircuts (recovery rates), to pass or to quit preemptive renegotiation—only available for

[6]Our model of debt restructuring with one-period bonds follows Benjamin and Wright (2009), Bi (2008), and Yue (2010). Relaxing the model to include long-duration bonds does not provide additional insights but increase technical difficulty to track the model. This is because old bonds are exchanged with new bonds with the same maturity and smaller outstanding (debt stock), i.e. no change in maturity structure of bonds due to an exchange (Hatchondo et al. 2014). See Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), Chatterjee and Eyingungor (2012) for long-duration bond models without debt restructurings.

[7]b_{max} exists when the interest rates on the sovereign's savings are sufficiently low compared to the discount factor, which is satisfied as $(1 + r^*)\beta < 1$.

[8]The direct productivity loss assumption in our production model is conceptually equivalent to "output costs" assumption in the conventional endowment model (e.g., Arellano 2008; Aguiar and Gopinath 2006; Yue 2010). In this regard, the direct production loss is widely accepted in the sovereign debt literature with endogenous production (Cuadra et al. 2010; Arellano and Bai 2017; Gordon and Guerron-Quintana 2018). Both assumptions are broadly in line with empirical estimates of output loss at default in general (Sturzenegger 2004; Tomz and Wright 2007; Levy-Yeyati and Panizza 2011) and those at a post-default restructuring (Asonuma and Trebesch 2016; Trebesch and Zabel 2017; Asonuma et al. 2019).

[9]Mendoza and Yue (2012) provide micro-foundation of this conventional assumption that exclusion from the international capital market leads to losses in production efficiency due to a lack of imported inputs and labor reallocation away from final goods production.

the preemptive case. The other party decides whether to accept, reject, or quit the proposal. If the offer with haircuts is proposed and accepted, then the sovereign regains full access to the international capital market in the next period and the creditors receive recovered debt payments. If one party quits the renegotiation, both parties move back to the sovereign's choice of passing preemptive option. Otherwise, both parties continue the negotiation over debt in the next period.

The sovereign has an incentive to settle on haircuts under both preemptive and post-default renegotiations: it opts to avoid prolonged partial exclusion from the market and smaller productivity loss for preemptive case and permanent exclusion from the market and larger productivity loss for post-default case. The creditors are willing to settle on recovery rates (haircuts) under post-default renegotiation because since renegotiation is the only option to recoup losses on the debt in arrear. They are also willing to settle under preemptive renegotiation if the recovery rates are at least higher than the expected return on bonds accounting for both the probability of full repayment and the recovery rates conditional on default.

3.3 Timing of the Model

Figure 5: Timing of Model

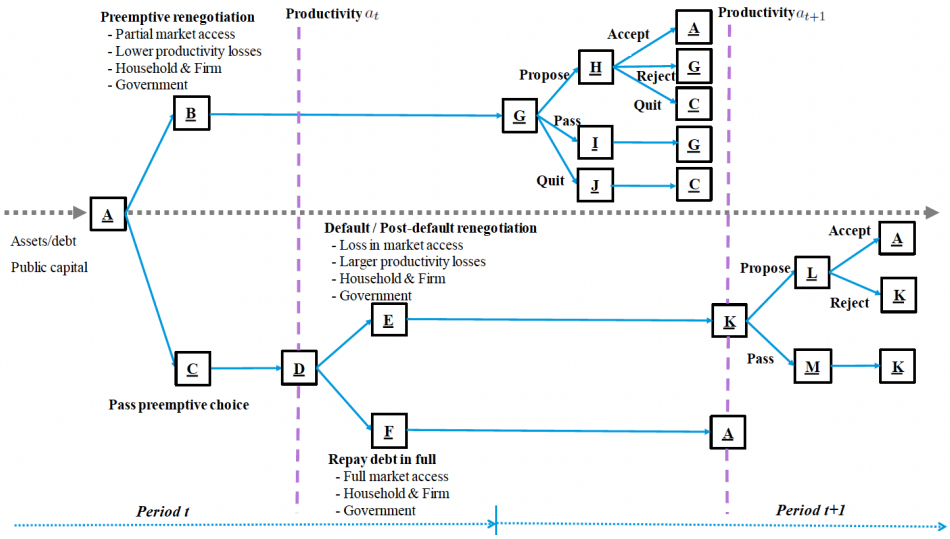


Figure 5 summarizes the timing of decisions within each period.

1. The sovereign starts current period with initial assets/debt, and public capital. We are in node (A).

2. The sovereign decides whether to initiate a preemptive debt restructuring or not.

   (a) If the sovereign opts a preemptive restructuring, we move to the upper branch of the tree and are in node (B). The sovereign has partial access to the market, suffers the smaller direct productivity loss, and chooses public expenditure. The household and firm make their decision. We move to node (G) in the next period.

   (b) If the sovereign passes the preemptive option, we move to the lower branch of the tree and are in node (C).

3. A productivity shock (a_t) realizes. The sovereign decides whether to pay its debt or to default.

   (a) In node (E) (default node), if it defaults, we move to the middle-upper branch of a tree. It loses access to the market, suffers the direct productivity loss, and chooses public expenditure. The household and firm make their decision. We proceed to note (K) in the next period.

   (b) In node (F) (repayment node), the sovereign repays debt, we move to the upper branch of a tree. It maintains market access and chooses assets/debt and expenditure. Foreign creditors choose sovereign bonds in the next period. The household and firm make their decision. We proceed to node (A) in the next period.

4. In node (G) (preemptive renegotiation), the proposer decides whether to propose an offer, pass or quit the renegotiation.

   (a) In node (H) (propose node), if the proposer proposes an offer, the counterpart decides whether to accept, reject or quit the renegotiation. If the counterpart accepts the offer, the sovereign regains market access and we move to node (A) in the next period. If the counterpart rejects the offer, the sovereign's market access remains partial and we move back to node (G) in the next period. If the counterpart quits, we proceed to node (C) in the current period.

   (b) In node (I) (pass node) if the proposer passes, the sovereign's market access remains partial in the next period and we move back to node (G).

   (c) In node (J) (quit node), if the proposer quits, we proceed to node (C) in the current period.

5. A productivity shock (a_{t+1}) realizes.

6. In node (K) (post-default renegotiation), the propose decides whether to propose an offer or pass.

7. (a) In node (L) (propose node), if the proposer chooses to propose an offer, the counterpart decides whether to accept or reject. If the counterpart accepts the offer, the sovereign regains market access and we move to node (A) in the next period. If the counterpart rejects, the sovereign remains in autarky and we move back to node (K) in the next period.

   (b) In node (M) (pass node) if the proposer chooses to pass, the sovereign remains in autarky and we move back to node (K) in the next period.

4 Recursive Equilibrium

4.1 Household's Problem

This section defines the stationary recursive equilibrium of our model. A representative household's utility function is defined as:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t [(1-\omega)u(c_t, l_t) + \omega v(g_t)]$$

where $0 < \beta < 1$ is the discount factor and c_t, l_t, g_t denote private consumption, labor supply and public consumption in period t, respectively. His period utility function is separable between a multiple of private consumption and labor supply, and public consumption. Both $u(\cdot)$ and $v(\cdot)$ are continuous, strictly increasing, strictly concave, and satisfy the Inada conditions. ω denotes the weight on public consumption in the household's utility function.

The household takes as given the wage rate w_t, profits paid by a firm π_t^F, public transfers T_t, public consumption g_t and a consumption tax rate τ, and chooses private consumption and labor supply. He does not borrow directly from abroad, but the government borrows, provides public consumption and transfers, and makes default decisions internalizing the household's utility. The household's optimization problem is written as

$$\begin{aligned} &\max E_0 \sum_{t=0}^{\infty} \beta^t [(1-\omega)u(c_t, l_t) + \omega v(g_t)] \\ &s.t. \quad (1+\tau)c_t = w_t l_t + \pi_t^F + T_t \end{aligned} \tag{1}$$

The consumption tax rate is assumed to be constant (Arellano and Bai 2017; Alfaro and Kanczuk 2017)—also supported by empirical findings on value-added taxes in developing countries in Gunter et al. (2017). The optimality condition of the household is shown as follows:

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{w_t}{1+\tau} \tag{2}$$

4.2 Firm's Problem

A representative firm chooses labor l_t for goods production given exogenous productivity shock a_t, public capital stock k_t^g, and fixed private capital stock $k^p(=1)$. The production function is Cobb-Douglas:

$$y_t = a_t(l_t)^{\alpha_l}(k_t^g)^{\alpha_k}(\bar{k_t^p})^{1-\alpha_l-\alpha_k} \tag{3}$$

where $0 < \alpha_l, \alpha_k < 1$. The fixed private capital stock assumption follows closely Mendoza and Yue (2012) and Azzimonti (2015) and Appendix C allows for the production function to have either decreasing or constant returns to scale, and shows that our main qualitative results remain robust. The firm's optimization problem is written as:

$$\max_{l_t} \pi_t^F = a_t(l_t)^{\alpha_l}(k_t^g)^{\alpha_k}(\bar{k_t^p})^{1-\alpha_l-\alpha_k} - w_t l_t \tag{4}$$

The optimality condition of the firm is shown as follows:

$$w_t = \alpha_l a_t(l_t)^{\alpha_l - 1}(k_t^g)^{\alpha_k}(\bar{k_t^p})^{1-\alpha_l-\alpha_k} \tag{5}$$

4.3 Sovereign's (Government's) Problem

The sovereign's problem is to maximize expected lifetime utility. We start the sovereign's problem with a good credit record ($h_t = 0$), i.e., full access to the market.

Prior to realization of current productivity, the sovereign decides whether it initiates a preemptive restructuring or not:

$$V^{EXANTE}(b_t, k_t^g, 0, a_{t-1}) = \max\left[V^{PRE}(b_t, k_t^g, 0, a_{t-1}), V^{NON\_PRE}(b_t, k_t^g, 0, a_{t-1})\right] \tag{6}$$

$V^{PRE}(b_t, k_t^g, 0, a_{t-1})$ is the value of initiating a preemptive restructuring:

$$V^{PRE}(b_t, k_t^g, 0, a_{t-1}) = \max_{g_t, b_{t+1}, k_{t+1}^g, T_t} \int_A [(1-\omega)u(c_t, l_t) + \omega v(g_t) + \beta\Psi(b_t, k_{t+1}^g, 1, a_t)]d\mu(a_t|a_{t-1}) \tag{7}$$

$$s.t. \quad g_t + k_{t+1}^g + T_t = \tau c_t + (1-\delta^k)k_t^g - \frac{\Omega}{2}(\frac{k_{t+1}^g - k_t^g}{k_t^g})^2 k_t^g \tag{8}$$

$$T_t \geq 0 \tag{9}$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l a_t(l_t)^{\alpha_l - 1}(k_t^g)^{\alpha_k}(\bar{k_t^p})^{1-\alpha_l-\alpha_k}}{1+\tau} \tag{10}$$

$$(1+\tau)c_t + k_{t+1}^p = \hat{y}_t + T_t + (1-\delta^k)k_t^p - \frac{\Omega}{2}(\frac{k_{t+1}^p - k_t^p}{k_t^p})^2 k_t^p \tag{11}$$

where $\Psi(b_t, k_t^g, 1, a_{t-1})$ represents value of preemptive debt renegotiation in the current period with an intermediate credit record ($h_t = 1$). $\hat{y}_t = \hat{a}_t(l_t)^{\alpha_l}(k_t^g)^{\alpha_k}(\bar{k_t^p})^{1-\alpha_l-\alpha_k}$ indicating output with small direct productivity loss ($\hat{a}_t$). Equation (8) is the budget constraint for the sovereign where it receives consumption tax revenues τc_t and public capital stock net of depreciation and adjustment costs $\frac{\Omega}{2}(\frac{k_{t+1}^p - k_t^p}{k_t^p})^2 k_t^p$—non-linear adjustment costs are assumed and δ^k is the depreciation rate of private and public capital—and allocates to public consumption g_t, capital k_{t+1}^g, transfers T_t. Equation (9) is the "non-lump sum taxation constraint" indicating a limitation of the government from transferring resources from the private sector. Mechanically, the government can freely transfer positive net borrowing through transfers, but cannot extract more resources from the private sector beyond the distortionary consumption tax revenues. Equations (10) and (11) denote the combined optimality condition for both the household and the firm and resource constraint respectively.

$V^{NON\_PRE}(b_t, k_t^g, 0, a_{t-1})$ is the sovereign's ex-ante value of postponing its preemptive restructuring, which is based on ex post value $V(b_t, k_t^g, 0, a_t)$ as follows:

$$V^{NON\_PRE}(b_t, k_t^g, 0, a_{t-1}) = \int_A V(b_t, k_{t+1}^g, 0, a_t) d\mu(a_t|a_{t-1}) \tag{12}$$

The sovereign's preemptive restructuring choice can be characterized by preemptive restructuring set $PRE(b_t, k_t^g, 0) \subset A$. It is a set of productivity shocks a_{t-1} which preemptive choice is optimal:

$$PRE(b_t, k_t^g, 0) = \{a_{t-1} \in A : V^{PRE}(b_t, k_t^g, 0, a_{t-1}) \geq V^{NON\_PRE}(b_t, k_t^g, 0, a_{t-1})\} \tag{13}$$

After realization of current productivity, the sovereign chooses whether to repay the debt or to default

$$V(b_t, k_t^g, 0, a_t) = \max\left[V^R(b_t, k_t^g, 0, a_t), V^D(b_t, k_t^g, 0, a_t)\right] \tag{14}$$

$V^R(b_t, k_t^g, 0, a_t)$ is its value associated with paying debt/saving:

$$V^R(b_t, k_t^g, 0, a_t) = \max_{g_t, b_{t+1}, k_{t+1}^g, T_t} (1-\omega)u(c_t, l_t) + \omega v(g_t) + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \tag{15}$$

$$s.t. \quad (9) \quad and \quad g_t + k_{t+1}^g + T_t + q(b_{t+1}, k_{t+1}^g, 0, a_t) b_{t+1} = \tau c_t + (1-\delta^k)k_t^g - \frac{\Omega}{2}(\frac{k_{t+1}^g - k_t^g}{k_t^g})^2 k_t^g + b_t \tag{8a}$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l a_t (l_t)^{\alpha_l - 1}(k_t^g)^{\alpha_k}(k_t^p)^{1-\alpha_l-\alpha_k}}{1+\tau} \tag{10a}$$

$$(1+\tau)c_t + k_{t+1}^g = y_t + T_t + (1-\delta^k)k_t^g - \frac{\Omega}{2}(\frac{k_{t+1}^g - k_t^g}{k_t^g})^2 k_t^g \tag{11a}$$

where b_t is saving/debt and $q(b_{t+1}, k^g_{t+1}, 0, a_t)b_{t+1}$ is assets/debt in the next period.

$V^D(b_t, k^g_t, 0, a_t)$ is its value associated with default:

$$V^D(b_t, k^g_t, 0, a_t) = \max_{g_t, k^g_{t+1}, T_t} (1-\omega)u(c_t, l_t) + \omega v(g_t) + \beta \int_A V((1+r^*)b_t, k^g_{t+1}, 1, a_{t+1}) d\mu(a_{t+1}|a_t) \tag{16}$$

$$s.t. \quad (8), \quad (9) \text{ and}$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l \tilde{a}_t (l_t)^{\alpha_l - 1} (k^g_t)^{\alpha_k} (k^p_t)^{1-\alpha_l-\alpha_k}}{1+\tau} \tag{10b}$$

$$(1+\tau)c_t + k^g_{t+1} = \tilde{y}_t + T_t + (1-\delta^k)k^g_t - \frac{\Omega}{2}(\frac{k^g_{t+1} - k^g_t}{k^g_t})^2 k^g_t \tag{11b}$$

where $\tilde{y}_t = \tilde{a}_t (l_t)^{\alpha_l} (k^g_t)^{\alpha_k} (\bar{k^p_t})^{1-\alpha_l-\alpha_k}$ indicating output with large direct productivity loss $(\tilde{a}_t)$.

The sovereign's default/post-default restructuring choice can be characterized by default set $D(b_t, k^g_t, 0) \subset A$ It is a set of productivity shocks a_t at which default is optimal:

$$D(b_t, k^g_t, 0) = \{a_t \in A : V^R(b_t, k^g_t, 0, a_t) < V^D(b_t, k^g_t, 0, a_t)\} \tag{17}$$

Next comes the country's problems with an intermediate credit record ($h_t = 1$) and a bad credit record ($h_t = 2$), (i.e., partial or no access to the market), respectively. Its value of continuing preemptive or post-default renegotiations is an expected payoff that the debtor obtains from the bargaining which starts from period t:

$$V(b_t, k^g_t, 1, a_{t-1}) = \Psi(b_t, k^g_t, a_{t-1}) \tag{18}$$

$$V(b_t, k^g_t, 2, a_t) = \Gamma(b_t, k^g_t, a_t) \tag{19}$$

4.4 Foreign Creditors' Problem

Foreign creditors are risk-neutral and can borrow from the international market with the risk-free rate (r^f). When the sovereign has a good credit record ($h_t = 0$)—also when it issues new debt at settlement (i.e., net issuance)—, given the sovereign bond price, foreign creditors choose the amount of assets/debt in the next period (b_{t+1}) to maximize the expected profit:

$$\pi^c(b_{t+1}, k^g_{t+1}, 0, a_t) = \begin{cases} q(b_{t+1}, k^g_{t+1}, 0, a_t) b_{t+1} - \frac{1}{1+r^*} b_{t+1}, & \text{if } b_{t+1} \geq 0 \\ \\ \frac{\delta(b_{t+1}, k^g_{t+1}, 1, a_t)}{1+r^*}(-b_{t+1}) & \text{if } b_{t+1} < 0 \text{ and} \\ -q(b_{t+1}, k^g_{t+1}, 0, a_t)(-b_{t+1}), & a_t \in PRE(b_{t+1}, k^g_{t+1}, 0) \\ \\ [\frac{1-p^D(b_{t+1}, k^g_{t+1}, 0, a_t)}{1+r^*} + & \\ \frac{p^D(b_{t+1}, k^g_{t+1}, 0, a_t) \int_A \gamma(b_{t+1}, k^g_{t+1}, 1, a_{t+1}) d\mu(a_{t+1}|a_t)}{1+r^*}](-b_{t+1}) & \\ -q(b_{t+1}, k^g_{t+1}, 0, a_t)(-b_{t+1}), & otherwise \end{cases} \tag{20}$$

where $\delta(b_{t+1}, k^g_{t+1}, 0, a_t)$, $p^D(b_{t+1}, k^g_{t+1}, 0, a_t)$, and $\alpha(b_{t+1}, k^g_{t+1}, 1, a_{t+1})$ are expected recovery rates at preemptive restructuring, expected default probability and expected recovery rates at post-default restructuring, respectively. Since we assume that the market for new sovereign bonds is completely competitive, foreign creditors' expected profit is zero in equilibrium. Using the zero expected profit condition, we get

$$q(b_{t+1}, k^g_{t+1}, 0, a_t) = \begin{cases} \frac{1}{1+r^*} & \text{if } b_{t+1} \geq 0 \\ \\ \frac{\delta(b_{t+1}, k^g_{t+1}, 1, a_t)}{1+r^*}(-b_{t+1}) & \text{if } b_{t+1} < 0 \text{ and} \\ & a_t \in PRE(b_{t+1}, k^g_{t+1}, 0) \\ \\ \frac{1-p^D(b_{t+1}, k^g_{t+1}, 0, a_t)}{1+r^*} & \\ + \frac{p^D(b_{t+1}, k^g_{t+1}, 0, a_t) \int_A \gamma(b_{t+1}, k^g_{t+1}, 1, a_{t+1}) d\mu(a_{t+1}|a_t)}{1+r^*} & otherwise \end{cases} \tag{21}$$

When the sovereign buys bonds from foreign creditors $b_{t+1} \geq 0$, the sovereign bond price is equal to the price of a risk-free bond, $\frac{1}{(1+r^*)}$. When the sovereign issues bonds to foreign creditors $b_{t+1} < 0$, the creditors face the risk of a default and post-default restructuring with expected recovery rates and the risk of a preemptive restructuring with no default but a haircut equivalent to 1 minus the recovery rate $\delta(b_{t+1}, k^g_{t+1}, 0, a_t)$. The sovereign bond is priced to compensate the creditors for these two types of risks. Since $0 \leq \gamma(b_{t+1}, k^g_{t+1}, 1, a_{t+1}) \leq 1$ and $0\alpha(b_{t+1}, k^g_{t+1}, 2, a_{t+1}) \leq 1$, the bond price $q(b_{t+1}, k^g_{t+1}, 0, a_t)$ lies in $[0, \frac{1}{(1+r^*)}]$..

4.5 Debt Renegotiation

Two types of debt renegotiations (preemptive and post-default) are symmetric in bargaining game and power. Both renegotiations take the form of a two-player stochastic bargaining game with complete information as in Merlo and Wilson (1995)[10]. It is a multi-round stochastic bargaining game in that both the productivity process of the sovereign debtor and the identity of the proposer are stochastic. For simplicity, each player has a constant probability of being selected as the proposer in each round of the negotiation. That is, the identity of the proposer is independent of the sovereign's productivity process. Let ϕ denote the probability that the borrower, B, can propose and $1-\phi$ is the probability that the lender, L, can propose. The probability which one of the players is selected as the proposer is a parsimonious way to reflect the bargaining power obtained through one's ability to enjoy the first-mover advantage.

Two types of debt renegotiations differ in three aspects: (i) timing, (ii) strategies and (ii) outside options for two parties. Preemptive renegotiation takes place before realization of current productivity, while post-default renegotiation after realization of current productivity and the sovereign chooses to default. For preemptive renegotiation, both parties have an option to quit renegotiation. Outside options under preemptive case are passing preemptive option for the sovereign and ex ante expected return on bonds for the creditors, while those under post-default case are permanent autarky for the sovereign and no recovered debt payments for the creditors.

In every round, the proposer may either propose recovery rates, pass or quit the renegotiation—only available for preemptive case. If he proposes, then the other party chooses to accept, reject the proposal or quit the renegotiation.[11] If the offer with recovery rates is proposed and accepted, the sovereign debtor repays the recovered debt payments and resumes full access to the market next period. If the proposer or the counterpart quit the renegotiation, both parties move back to the sovereign's choice of passing preemptive option. Otherwise (i.e. when the proposer proposes and the counterpart rejects or the proposes passes), both parties repeat the bargaining game in the next period.

We define some basic concepts of the game. A stochastic bargaining game may be denoted by $(C, \beta, 1/(1+r^*))$ where for each productivity process $a \in A$, $C(a)$ is the set of feasible utility vectors that may be agreed upon in that state. β and $1/(1+r^*)$ are the discount factors for B and L, respectively.[12] A payoff function is an element $\Delta(a) \in C(a)$, where $\Delta_i(a)$ is the utility to player i for $i = B, L$.

As in Merlo and Wilson (1995), we focus on a game with stationary strategies, that is, the

[10] While there could be other approaches of modeling a bargaining game between two parties, we follow the conventional bargaining game in Merlo and Wilson (1995) for their simplicity and tractability.

[11] We assume that the proposer makes an offer that the respondent accepts when the value of proposing is higher or equal to the value of passing, and chooses to pass otherwise. This assumption can get rid of trivial source of multiplicity. See Merlo and Wilson (1995) and Ortner (2013) for the same treatment.

[12] Merlo and Wilson (1995) assume that the players have the same discount factor. But they also explain that "there is no real restriction implied by the assumption that players discount utility at a common constant rate. So long as the discounted size of the "cake" converges uniformly to 0. $\cdots$ player-dependent discount factors can always be represented by a different "cake" process with a common fixed discount factor". So in our model, we assume that the borrower and the lender have different discount factors.

players' actions depend only on the current state $(b_t, k_t^g, 1, a_t)$ where $h_t = 1, 2$ and the current offer. In equilibrium, the proposer's strategy is to propose when the other party would accept for sure and to pass otherwise. In contrast, the other player's strategy is to accept when the proposal is made, to reject when the proposer passes, and to quit otherwise. Therefore, we can denote the proposer i's and the other party j's equilibrium strategies as follows: for $h_t = 1, 2$, (a) $\theta_i(b_t, k_t^g, 1, a_t) = 1$ (propose) when the proposer i proposes and $\theta_j(b_t, k_t^g, 1, a_t) = 1$ (accept) when the other party$j$ accepts the offer, (b) $\theta_i(b_t, k_t^g, 1, a_t) = 0$ (pass) when the proposer i passes and

$$\theta_j(b_t, k_t^g, 1, a_t) = 0$$

(reject) when the other party j rejects the offer, or only for $h_t = 1$, (c) $\theta_i(b_t, k_t^g, h_t, a_t) = -1$ (quit) when the proposer i quits the renegotiation and $\theta_j(b_t, k_t^g, h_t, a_t) = -1$ (quit) when the other party j quits.[13]

A stationary subgame perfect (SP) equilibrium is defined as the players' equilibrium stationary strategies θ and θ^*, and the payoff functions, Γ and Γ^*, associated with these strategies for player B and L. The expected payoff at preemptive and post-default debt renegotiations for the borrower B and lender L in period t, are shown as:

$$\begin{aligned}\Gamma(b_t, k_t^g, 2, a_t) &= \phi\Gamma^B(b_t, k_t^g, 2, a_t) + (1-\phi)\Gamma^L(b_t, k_t^g, 2, a_t)\\ \Gamma^*(b_t, k_t^g, 2, a_t) &= \phi\Gamma^{*B}(b_t, k_t^g, 2, a_t) + (1-\phi)\Gamma^{*L}(b_t, k_t^g, 2, a_t)\end{aligned} \tag{22}$$

$$\begin{aligned}\Psi(b_t, k_t^g, 1, a_{t-1}) &= \phi\Psi^B(b_t, k_t^g, 1, a_{t-1}) + (1-\phi)\Psi^L(b_t, k_t^g, 1, a_{t-1})\\ \Psi^*(b_t, k_t^g, 1, a_{t-1}) &= \phi\Psi^{*B}(b_t, k_t^g, 1, a_{t-1}) + (1-\phi)\Psi^{*L}(b_t, k_t^g, 1, a_{t-1})\end{aligned} \tag{23}$$

Here the superscript denotes the identity of the propose: $\Gamma^B(\Gamma^{*B})$ represents the borrower's (lender's) payoff when the borrower is the proposer and $\Gamma^L(\Gamma^{*L})$ refers to the borrower's (lender's) payoff when the lender is the proposer.

First, we start with post-default debt renegotiation. We consider the case when the borrower B is the proposer. We denote the proposed debt recovery rates as α_t^B, the borrower's values of proposing and passing as V^{PRO} and V^{PASS}, and the creditors' values of accepting offer and rejecting as V^{*ACT} and V^{*REJ}, respectively. When the borrower B proposes and the proposal is accepted, the sovereign repays reduced debt arrears $-\alpha_t^B b_t$ and resumes access to the market in the next period with outstanding debt, i.e., net issuance at the settlement as in Benjamin and Wright (2013). Appendix D assumes no net issuance and shows that our main qualitative and quantitative results remain robust.

[13]Benjamin and Wright (2009) theoretically prove both existence and uniqueness of the equilibrium in multi-round bargaining game over defaulted debt.

$$V^{PRO}(b_t, k_t^g, 2, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1-\omega)u(c_t, l_t) + \omega v(g_t) + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \tag{24}$$

$$s.t. \quad (9),\ (10a),\ (11a),\ \text{and}$$

$$g_t + k_{t+1}^g + T_t = \tau c_t + (1-\delta^k)k_t^g - \frac{\Omega}{2}(\frac{k_{t+1}^g - k_t^g}{k_t^g})^2 k_t^g + \delta_t^B b_t \tag{8b}$$

$$V^{*ACT}(b_t, k_t^g, a_t) = -\delta_t^B b_t \tag{25}$$

When the borrower B passes, both parties proceed to the next period with accumulated arrears.

$$V^{PASS}(b_t, k_t^g, 2, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1-\omega)u(c_t, l_t) + \omega v(g_t) + \beta \int_A V((1+r^*)b_t, k_{t+1}^g, 2, a_{t+1}) d\mu(a_{t+1}|a_t) \tag{26}$$

$$s.t. \quad (8),\ (9),\ (10b),\ \text{and}\ (11b)$$

$$V^{*REJ}(b_t, k_t^g, 2, a_t) = \frac{1}{1+r^*} \int_A \Gamma^*((1+r^*)b_t, k_{t+1}^g, 2, a_{t+1}) d\mu(a_{t+1}|a_t) \tag{27}$$

In equilibrium, the agreed recovery rates δ_t^{B*} satisfy the following:[14]

$$\begin{aligned} \delta_t^{B*} &= argmax V^{PRO}(b_t, k_t^g, 2, a_t) \\ s.t. \quad V^{PRO}(b_t, k_t^g, 2, a_t) &\geq V^{PASS}(b_t, k_t^g, 2, a_t) \\ V^{*ACT}(b_t, k_t^g, 2, a_t) &\geq V^{*REJ}(b_t, k_t^g, 2, a_t) \end{aligned} \tag{28}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\begin{aligned} \Gamma^B(b_t, k_t^g, 2, a_t) &= V^{PRO}(b_t, k_t^g, 2, a_t) \\ \Gamma^{B*}(b_t, k_t^g, 2, a_t) &= V^{*ACT}(b_t, k_t^g, 2, a_t) \end{aligned} \tag{29}$$

Otherwise,

$$\Gamma^B(b_t, k_t^g, 2, a_t) = V^{PASS}(b_t, k_t^g, 2, a_t)$$

[14]Off-equilibrium paths are eliminated in equilibrium.

$$\Gamma^{B*}(b_t, k_t^g, 2, a_t) = V^{*REJ}(b_t, k_t^g, 2, a_t) \tag{27a}$$

The renegotiation settlement can be characterized by settlement set $R^B(b_t, k_t^g, 2) \subset A$. It is a set of productivity shocks a_t at which both parties reach an agreement:

$$R^B(b_t, k_t^g, 2) = \left\{ \begin{array}{r} a_t \in A : V^{PRO}(b_t, k_t^g, 2, a_t) \geq V^{PASS}(b_t, k_t^g, 2, a_t) \\ V^{*ACT}(b_t, k_t^g, 2, a_t) \geq V^{*REJ}(b_t, k_t^g, 2, a_t) \end{array} \right\} \tag{30}$$

Similarly, when the lender is the proposer, we denote the proposed debt recovery rate as α_t^L, the borrower's value as V^{ACT} and V^{REJ}, and the lender's values as V^{*PRO} and V^{*PASS}, respectively. When the borrower L proposes and the proposal is accepted,

$$V^{*PRO}(b_t, k_t^g, 2, a_t) = -\delta_t^L b_t \tag{31}$$

$$V^{PRO}(b_t, k_t^g, 2, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1-\omega)u(c_t, l_t) + \omega v(g_t) + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \tag{32}$$

$$s.t. \quad (9),\ (10b),\ (11b),\ \text{and}$$

$$q(b_{t+1}, k_{t+1}^g, 0, a_t b_{t+1}) + g_t + k_{t+1}^g + T_t = \tau c_t + (1-\delta^k)k_t^g - \frac{\Omega}{2}(\frac{k_{t+1}^g - k_t^g}{k_t^g})^2 k_t^g + \delta_t^B b_t \tag{8c}$$

When the lender passes,

$$V^{*PASS}(b_t, k_t^g, 2, a_t) = \frac{1}{1+r^*} \int_A \Gamma^*((1+r^*)b_t, k_{t+1}^g, 2, a_{t+1}) d\mu(a_{t+1}|a_t) \tag{33}$$

$$V^{REJ}(b_t, k_t^g, 2, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1-\omega)u(c_t, l_t) + \omega v(g_t) + \beta \int_A V((1+r^*)b_t, k_{t+1}^g, 2, a_{t+1}) d\mu(a_{t+1}|a_t) \tag{34}$$

$$s.t. \quad (8),\ (9),\ (10b),\ \text{and}\ (11b)$$

In equilibrium, the agreed recovery rates δ_t^{L*} satisfy the following:

$$\begin{aligned} \delta_t^{L*} &= argmax V^{*PRO}(b_t, k_t^g, 2, a_t) \\ s.t. \quad & V^{*PRO}(b_t, k_t^g, 2, a_t) \geq V^{*PASS}(b_t, k_t^g, 2, a_t) \\ & V^{ACT}(b_t, k_t^g, 2, a_t) \geq V^{REJ}(b_t, k_t^g, 2, a_t) \end{aligned} \tag{26a}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\Gamma^{*L}(b_t, k_t^g, a_t) = V^{*PRO}(b_t, k_t^g, a_t)$$
$$\Gamma^{L}(b_t, k_t^g, a_t) = V^{ACT}(b_t, k_t^g, a_t) \tag{27b}$$

Otherwise,

$$\Gamma^{*L}(b_t, k_t^g, a_t) = V^{*PASS}(b_t, k_t^g, a_t)$$
$$\Gamma^{L}(b_t, k_t^g, a_t) = V^{REJ}(b_t, k_t^g, a_t) \tag{27c}$$

The renegotiation settlement can be characterized by settlement set $R^L(b_t, k_t^g, 2) \subset A$. It is a set of productivity shocks a_t at which both parties reach an agreement:

$$R^L(b_t, k_t^g, 2) = \left\{ \begin{array}{c} a_t \in A : V^{*PRO}(b_t, k_t^g, 2, a_t) \geq V^{*PASS}(b_t, k_t^g, 2, a_t) \\ V^{ACT}(b_t, k_t^g, 2, a_t) \geq V^{REJ}(b_t, k_t^g, 2, a_t) \end{array} \right\}. \tag{28a}$$

Second, we consider preemptive debt renegotiation. We consider the case when the borrower B is the proposer. We denote the proposed debt recovery rates as δ_t^B, the borrower's values of proposing, passing, and quitting as V^{PRO}, V^{PASS}, and V^{QUIT}, the creditors' values of accepting, rejecting, and quitting as V^{*ACT}, V^{*REJ}, and V^{*QUIT} respectively. When the borrower B proposes and the proposal is accepted, the sovereign pays agreed debt repayments $-\delta_t^B b_t$ and resumes access to the market in the next period with outstanding debt:

$$V^{PRO}(b_t, k_t^g, 1, a_t) = \max_{g_t, k_{t+1}^g, T_t} \int_A (1-\omega)u(c_t, l_t) + \omega v(g_t) + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \tag{35}$$

$$s.t. \quad (9),\ (10b),\ (11b),\ \text{and}$$

$$q(b_{t+1}, k_{t+1}^g, 0, a_t b_{t+1}) + g_t + k_{t+1}^g + T_t = \tau c_t + (1-\delta^k)k_t^g - \frac{\Omega}{2}(\frac{k_{t+1}^g - k_t^g}{k_t^g})^2 k_t^g + \delta_t^B b_t \tag{8d}$$

$$V^{*ACT}(b_t, k_t^g, 1, a_{t-1}) = -\delta_t^B b_t \tag{36}$$

When the borrower B passes, both parties proceed to the next period with the same value of debt.

$$V^{PASS}(b_t, k_t^g, 1, a_{t-1}) = \max_{g_t, k_{t+1}^g, T_t} \int_A \left[\begin{array}{c} (1-\omega)u(c_t, l_t) + \omega v(g_t) + \\ \beta \Psi(b_{t+1}, k_{t+1}^g, 1, a_t) \end{array} \right] d\mu(a_{t+1}|a_t) \tag{37}$$

$$s.t. \quad (8),\ (9),\ (10),\ \text{and},\ (11)$$

$$V^{*REJ}(b_t, k_t^g, 1, a_{t-1}) = \frac{1}{1+r^*} \int_A \Psi^*((1+r^*)b_t, k_{t+1}^g, 1, a_t) d\mu(a_{t+1}|a_t) \tag{38}$$

When the borrower B quits the preemptive debt renegotiation, the sovereign proceeds to its choice between repayment and default without any debt treatment and the foreign creditors receive expected return on sovereign bonds:

$$V^{QUIT}(b_t, k_t^g, 1, a_{t-1}) = V^{NON-PRE}(b_t, k_t^g, 0, a_{t-1}) \tag{39}$$

$$V^{*QUIT}(b_t, k_t^g, 1, a_{t-1}) = (1 - p^D(b_t, k_t^g, 0, a_{t-1})) + p^D(b_t, k_t^g, 0, a_{t-1})\gamma(b_t, k_t^g, 2, a_{t-1}) \tag{40}$$

In equilibrium, the agreed recovery rates δ_t^{B*} satisfy the following:[15]

$$\begin{aligned} \delta_t^{B*} &= argmax V^{PRO}(b_t, k_t^g, 1, a_t) \\ s.t. \quad V^{PRO}(b_t, k_t^g, 1, a_{t-1}) &\geq V^{PASS}(b_t, k_t^g, 1, a_{t-1}) \\ V^{PRO}(b_t, k_t^g, 1, a_{t-1}) &\geq V^{QUIT}(b_t, k_t^g, 1, a_{t-1}) \\ V^{*ACT}(b_t, k_t^g, 1, a_{t-1}) &\geq V^{*REJ}(b_t, k_t^g, 1, a_{t-1}) \\ V^{*ACT}(b_t, k_t^g, 1, a_{t-1}) &\geq V^{*QUIT}(b_t, k_t^g, 1, a_{t-1}) \end{aligned} \tag{41}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\begin{aligned} \Psi^B(b_t, k_t^g, 1, a_{t-1}) &= V^{PRO}(b_t, k_t^g, 1, a_{t-1}) \\ \Psi^{*B}(b_t, k_t^g, 1, a_{t-1}) &= V^{*ACT}(b_t, k_t^g, 1, a_{t-1}) \end{aligned} \tag{42}$$

Otherwise,

$$\begin{aligned} \Psi^B(b_t, k_t^g, 1, a_{t-1}) &= V^{PASS}(b_t, k_t^g, 1, a_{t-1}) \\ \Psi^{*B}(b_t, k_t^g, 1, a_{t-1}) &= V^{*REJ}(b_t, k_t^g, 1, a_{t-1}) \end{aligned} \tag{43}$$

or

$$\begin{aligned} \Psi^B(b_t, k_t^g, 1, a_{t-1}) &= V^{QUIT}(b_t, k_t^g, 1, a_{t-1}) \\ \Psi^{*B}(b_t, k_t^g, 1, a_{t-1}) &= V^{*QUIT}(b_t, k_t^g, 1, a_{t-1}) \end{aligned} \tag{41a}$$

The renegotiation settlement can be characterized by settlement set $R^B(b_t, k_t^g, 1) \subset A$, defined as the set of productivity shocks a_{t-1} which both parties agree on settlements:

$$R^B(b_t, k_t^g, 1) = \left\{ \begin{array}{c} a_{t-1} \in A : V^{PRO}(b_t, k_t^g, 1, a_t) \geq V^{PASS}(b_t, k_t^g, 1, a_t) \\ V^{*ACT}(b_t, k_t^g, 1, a_t) \geq V^{*REJ}(b_t, k_t^g, 1, a_t) \end{array} \right\} \tag{44}$$

[15] Off-equilibrium paths are eliminated in equilibrium.

We consider the case when the borrower B is the proposer. We denote the proposed debt recovery rates as δ_t^B, the borrower's values of proposing, passing, and quitting as V^{PRO}, V^{PASS}, and V^{QUIT}, the creditors' values of accepting, rejecting, and quitting as V^{*ACT}, V^{*REJ}, and V^{*QUIT} respectively. When the borrower B proposes and the proposal is accepted, the sovereign pays agreed debt repayments $-\delta_t^B b_t$ and resumes access to the market in the next period with outstanding debt:

Similarly, when the lender is the proposer, we denote the proposed debt recovery rate as δ_t^L, the lender's values of proposing, passing and quitting as V^{*PRO}, V^{*PASS}, and V^{*QUIT}, the borrower's values of accepting, rejecting and quitting as V^{ACT}, V^{REJ}, V^{QUIT}. When the lender L proposes,

$$V^{*PRO}(b_t, k_t^g, 1, a_{t-1}) = -\delta_t^L b_t \tag{34a}$$

$$V^{ACT}(b_t, k_t^g, 1, a_{t-1}) = \max_{g_t, k_{t+1}^g, T_t} \int_A \left[\begin{array}{c} (1-\omega)u(c_t, l_t) + \omega v(g_t) + \\ \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \end{array} \right] d\mu(a_{t+1}|a_t) \tag{33a}$$

$$s.t. \quad (9),\ (10b),\ (11),\ \text{and}$$

$$q(b_{t+1}, k_{t+1}^g, 0, a_t b_{t+1}) + g_t + k_{t+1}^g + T_t = \tau c_t + (1-\delta^k) k_t^g - \frac{\Omega}{2}\left(\frac{k_{t+1}^g - k_t^g}{k_t^g}\right)^2 k_t^g + \delta_t^B b_t \tag{8e}$$

$$V^{ACT}(b_t, k_t^g, 1, a_{t-1}) \geq V^{NON-PRE}(b_t, k_t^g, 0, a_{t-1}) \tag{34b}$$

When the lender passes,

$$V^{*PASS}(b_t, k_t^g, 1, a_{t-1}) = \frac{1}{1+r^*} \int_A \Psi^*(b_t, k_{t+1}^g, 1, a_t) d\mu(a_{t+1}|a_t) \tag{36a}$$

$$V^{REJ}(b_t, k_t^g, 1, a_{t-1}) = \max_{g_t, k_{t+1}^g, T_t} \int_A \left[\begin{array}{c} (1-\omega)u(c_t, l_t) + \omega v(g_t) + \\ \beta \int_A \Psi^B(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \end{array} \right] d\mu(a_{t+1}|a_t) \tag{35a}$$

$$s.t. \quad (8),\ (9),\ (10),\ \text{and},\ (11)$$

When the lender quits the preemptive debt renegotiation,

$$V^{*QUIT}(b_t, k_t^g, 1, a_{t-1}) = (1 - p^D(b_t, k_t^g, 0, a_{t-1})) + p^D(b_t, k_t^g, 0, a_{t-1}) \gamma(b_t, k_t^g, 2, a_{t-1}) \tag{38a}$$

$$V^{QUIT}(b_t, k_t^g, 1, a_{t-1}) = V^{NON-PRE}(b_t, k_t^g, 0, a_{t-1}) \tag{37a}$$

In equilibrium, agreed recovery rates δ_t^{L*} satisfy the following:

$$\begin{aligned} \delta_t^{L*} &= argmax V^{PRO}(b_t, k_t^g, 1, a_t) \\ s.t. \quad V^{*PRO}(b_t, k_t^g, 1, a_{t-1}) &\geq V^{*PASS}(b_t, k_t^g, 1, a_{t-1}) \\ V^{*PRO}(b_t, k_t^g, 1, a_{t-1}) &\geq V^{*QUIT}(b_t, k_t^g, 1, a_{t-1}) \\ V^{ACT}(b_t, k_t^g, 1, a_{t-1}) &\geq V^{REJ}(b_t, k_t^g, 1, a_{t-1}) \\ V^{ACT}(b_t, k_t^g, 1, a_{t-1}) &\geq V^{QUIT}(b_t, k_t^g, 1, a_{t-1}) \end{aligned} \tag{39a}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\begin{aligned} \Psi^{*L}(b_t, k_t^g, 1, a_{t-1}) &= V^{*PRO}(b_t, k_t^g, 1, a_{t-1}) \\ \Psi^{L}(b_t, k_t^g, 1, a_{t-1}) &= V^{ACT}(b_t, k_t^g, 1, a_{t-1}) \end{aligned} \tag{40a}$$

Otherwise,

$$\begin{aligned} \Psi^{*L}(b_t, k_t^g, 1, a_{t-1}) &= V^{*PASS}(b_t, k_t^g, 1, a_{t-1}) \\ \Psi^{L}(b_t, k_t^g, 1, a_{t-1}) &= V^{REJ}(b_t, k_t^g, 1, a_{t-1}) \end{aligned} \tag{41b}$$

or

$$\begin{aligned} \Psi^{*L}(b_t, k_t^g, 1, a_{t-1}) &= V^{*QUIT}(b_t, k_t^g, 1, a_{t-1}) \\ \Psi^{L}(b_t, k_t^g, 1, a_{t-1}) &= V^{REJ\_QUIT}(b_t, k_t^g, 1, a_{t-1}) \end{aligned} \tag{41c}$$

The renegotiation settlement can be characterized by settlement set $R^L(b_t, k_t^g, 1) \subset A$, defined as the set of productivity shocks a_{t-1} which both parties agree on settlements:

$$R^L(b_t, k_t^g, 1) = \left\{ \begin{array}{c} a_{t-1} \in A : V^{*PRO}(b_t, k_t^g, 1, a_t) \geq V^{*PASS}(b_t, k_t^g, 1, a_t) \\ V^{ACT}(b_t, k_t^g, 1, a_t) \geq V^{REJ}(b_t, k_t^g, 1, a_t) \end{array} \right\} \tag{42a}$$

4.6 Equilibrium

A recursive equilibrium is defined as a set of functions for (a) the sovereign's ex-ante and ex post value functions, public consumption, capital, transfers, assets/debt, sets of preemptive restructuring and default, (b) the household's private consumption and labor supply, (c) the firm's labor demand and private capital, (d) the sovereign's and the foreign creditors' decision functions, payoffs, recovery rates, settlement sets (all depending on who is the proposer), (e) sovereign bond price and wage such that

[1]. sovereign government 's value function, public consumption, capital, transfers, assets/debt position and default set satisfy its optimization problem (6)–(19);

[2]. the household's consumption and labor supply satisfy his optimazation problem (1)–(2);

[3]. the firm's labor demand and wage satisfies his optimzation problem (3)–(5);

[4]. the foreign creditors' assets and bond prices satisfy their optimization problem (20)–(21);

[5]. both parties' decisions, payoffs and recovery rates solve the multi-round preemptive and post-default debt renegotiation problems (22)–(44).

In equilibrium, default probability is defined by using the sovereign's default set:

$$p^D(b_{t+1}, k^g_{t+1}, 0, a_t) = \int_{D(b_{t+1}, k^g_{t+1}, 0)} d\mu(a_{t+1}|a_t), \tag{45}$$

$$p^{POST}(b_{t+1}, k^g_{t+1}, a_t) = \phi \int_{R^B(b_{t+1}, k^g_{t+1}, 2)} d\mu(a_{t+1}|a_t) + (1-\phi) \int_{R^L(b_{t+1}, k^g_{t+1}, 2)} d\mu(a_{t+1}|a_t),$$

$$p^{PRE}(b_{t+1}, k^g_{t+1}, a_t) = \phi \int_{R^B(b_{t+1}, k^g_{t+1}, 1)} d\mu(a_{t+1}|a_t) + (1-\phi) \int_{R^L(b_{t+1}, k^g_{t+1}, 1)} d\mu(a_{t+1}|a_t), \tag{46}$$

Expected recovery rates conditional on the sovereign's preemptive and post-default restructuring choice are shown as:

$$\delta(b_{t+1}, k^g_{t+1}, 1, a_t) = \int_A \left[\begin{array}{c} \phi \mathbb{1}_{a_{t+1} \in R^B(b_{t+2}, k^g_{t+2}, 1)} \delta^{B*}(b_{t+2}, k^g_{t+2}, a_{t+1}) \\ +(1-\phi) \mathbb{1}_{a_{t+1} \in R^L(b_{t+2}, k^g_{t+2}, 1)} \delta^{L*}(b_{t+2}, k^g_{t+2}, a_{t+1}) \\ + \left(\begin{array}{c} \phi \mathbb{1}_{a_{t+1} \notin R^B(b_{t+2}, k^g_{t+2}, 1)} \\ +(1-\phi) \mathbb{1}_{a_{t+1} \notin R^L(b_{t+2}, k^g_{t+2}, 1)} \end{array} \right) \delta(b_{t+2}, k^g_{t+2}, a_{t+1}) \end{array} \right] d\mu(a_{t+1}|a_t)$$

$$\alpha(b_{t+1}, k^g_{t+1}, 2, a_{t+1}) = \int_A \left[\begin{array}{c} \phi \mathbb{1}_{a_{t+2} \in R^B(b_{t+2}, k^g_{t+2}, 2)} \alpha^{B*}((1+r^*)b_{t+2}, k^g_{t+2}, a_{t+2}) \\ +(1-\phi) \mathbb{1}_{a_{t+2} \in R^L(b_{t+2}, k^g_{t+2}, 2)} \alpha^{L*}((1+r^*)b_{t+2}, k^g_{t+2}, a_{t+2}) \\ + \left(\begin{array}{c} \phi \mathbb{1}_{a_{t+2} \notin R^B(b_{t+2}, k^g_{t+2}, 2)} \\ +(1-\phi) \mathbb{1}_{a_{t+2} \notin R^L(b_{t+2}, k^g_{t+2}, 2)} \end{array} \right) \alpha((1+r^*)b_{t+2}, k^g_{t+2}, a_{t+2}) \end{array} \right] d\mu(a_{t+2}|a_{t+1}) \tag{47}$$

The sovereign's total spread, i.e., the difference between the sovereign's interest rate and the risk-free rate, is defined as

$$s(b_{t+1}, k^g_{t+1}, 0, a_t) = \frac{1}{q(b_{t+1}, k^g_{t+1}, 0, a_t)} - (1+r^*)$$

5 Quantitative Analysis

This section provides the quantitative analysis of our model applied to the Argentine default and post-default restructuring in 2001–05 and Uruguay preemptive restructuring in 2003. We obtain three key results. First, we predict the government's choice of ex ante and ex post expenditure consolidation. Our model finds that the government is more likely to take ex ante expenditure consolidation when it is certain on a future default following deterioration in productivity. Second, we predict that ex ante expenditure consolidation is followed by a preemptive restructuring, while ex post expenditure consolidation comes upon a default and post-default restructuring. Third, our simulation exercise successfully replicates the five stylized facts as observed in the data.

5.1 Parameters and Functional Forms

For our quantitative analysis, we take a conventional approach and follow parameter values and functional forms used in the literature of sovereign debt and fiscal policy. The household utility is constant relative risk aversion (CRRA) and follows Greenwood et al. (1998) function with no wealth effects on labor supply—the marginal rate of substitution between labor and private consumption does not change depending on the size of private consumption:

$$u(c_t, l_t) = \frac{(c_t - \frac{l_t^{1+\psi}}{1+\psi})^{1-\sigma}}{1-\sigma}, \qquad v(g_t) = \frac{g_t^{1-\sigma_g}}{1-\sigma_g} \tag{48}$$

Risk aversion for private and public consumption is identical $\sigma = \sigma_g = 3$ (Cuadra et al. 2010, Arellano and Bai 2017; Hatchondo et al. forthcoming). The government' appetite to smooth utility through two types of consumption remain the same. We set the risk-free interest as quarterly interest rate on the 3-month US Treasury bills $r^* = 0.01$ as in Aguiar et al. (2016). Labor elasticity is assumed as 0.48 as in Mendoza (1991). Labor and public capital income share is set as 0.64 and 0.058 for Argentina in 1993–2005 and 0.58 and 0.11 for Uruguay in 1993–2003. We assume depreciation rate on public capital as 0.04 as in US BEA (1999). Tax on consumption goods reflects effective rate computed based on total tax revenues for Argentina and Uruguay in IMF WEO as 0.33 (Argentina) and 0.21 (Uruguay).

The productivity is assumed to follow a log normal AR(1) process and its shock takes $i.i.d.$ $N(0, \sigma^{a,2})$ as in Gordon and Guerron-Quintana (2018):

$$\log(a_t) = \rho \log(a_{t-1}) + \epsilon_{a,t} \tag{49}$$

We calibrate the productivity to fit quarterly seasonally adjusted GDP data from the Ministry of Economy and Production in Argentina (MECON) and the Central Bank of Uruguay (CBU). Calibrated auto-correlation and standard deviation of the productivity shock are $\rho = 0.85$ and $\sigma^{a,2} = 0.017$ for Argentina and $\rho = 0.90$ and $\sigma^{a,2} = 0.015$ for Uruguay, respectively. Obtained stochastic process is approximated as a discrete Markov chain of equally spaced grids

Table 3: Model Parameters

Parameter	Value	Source
Risk aversion for private consumption	$\sigma = 3$	Hatchondo et al. (forthcoming)
Risk aversion for public consumption	$\sigma_g = 3$	Hatchondo et al. (forthcoming)
Labor elasticity	$\psi = 0.48$	Mendoza (1991)
Risk-free interest rate	$r^* = 0.01$	Aguiar et al. (2016), Yue (2010) - US Treasury bill rate
Public capital depreciation rate	$\delta^k = 0.04$	US BEA (1999)
Direct productivity loss (post-default)	$\lambda_d = 0.05$	Asonuma and Trebesch (2016) - Computed (ARG)
Direct productivity loss (preemptive)	$\lambda_p = 0.04$	Asonuma and Trebesch (2016) - Computed (URY)
Country-specific parameters		
Weight on public consumption	$\omega = 0.80$ (ARG)/0.80 (URY)	Computed (ARG/URY)
Labor income share	$\alpha^l = 0.64$ (ARG)/0.58 (URY)	Gordon and Guerron-Quintana (ARG)/Computed (URY)
Public capital income share	$\alpha^k = 0.058$ (ARG)/0.11 (URY)	Computed (ARG/URY)
Effective consumption tax rate	$\tau = 0.33$ (ARG)/0.33 (URY)	Computed - IMF WEO (ARG/URY)
Public capital adjustment costs	$\Omega = 10$ (ARG)/10 (URY)	Computed (ARG/URY)
Auto-correlation of productivity shock	$\rho = 0.85$ (ARG) /0.90 (URY)	Computed - MECON (ARG)/ BCU (URY)
Standard deviation of productivity shock	$\sigma^a = 0.017$ (ARG) /0.015 (URY)	Computed - MECON (ARG)/ BCU (URY)
Bargaining power	$\phi = 0.93$ (ARG)/0.70 (URY)	Computed (ARG/URY)
Discount rate	$\beta = 0.80$ (ARG)/0.80 (URY)	Computed (ARG/URY)

by applying Tauchen (1989)'s quadrature approach.

The direct productivity losses during two types of restructurings are assumed to take a conventional asymmetric output costs (Arellano 2008)

$$\tilde{a}_t = \begin{cases} (1-\lambda_d)E(a_t) & \text{if } a_t \geq (1-\lambda_d)E(a_t) \\ a_t & otherwise \end{cases}$$

$$\hat{a}_t = \begin{cases} (1-\lambda_p)E(a_t) & \text{if } a_t \geq (1-\lambda_p)E(a_t) \\ a_t & otherwise \end{cases} \tag{50}$$

where we set $\lambda_d = 0.05$, $\lambda_p = 0.04$ to replicate average GDP deviation from the trend during restructurings for Argentina and Uruguay. Calibrated output deviation from the trend is consistent with empirical evidence of lower output costs in preemptive restructurings (Asonuma and Trebesch 2016).

The weight on public consumption in the household's utility, and public capital adjustment costs are set as $\omega = 0.8$ and $\Omega = 10$ for Argentina and $\omega = 0.8$ and $\Omega = 10$ for Uruguay to replicate average public consumption and transfers-to-GDP ratio of 20.0% for Argentina and 19.4% for Uruguay and standard deviation of public investment relative to that of output of 5.1 for Argentina and of 5.8 for Uruguay, respectively.

Sturzenegger and Zettelmeyer (2006) report that Argentina experienced 6 restructurings and Uruguay experienced 6 restructurings in 1820-2004. Moreover, Struzenegger and Zettlemeyer (2008) show that the recovery rate (haircut) in Argentina 2001–05 post-default restructuring and

Uruguayan 2003 preemptive restructuring were 25.0% (75.0%) and 87.1% (12.9%), respectively. We specify the sovereign's discount factor and bargaining power $\beta = 0.80$, $\phi = 0.93$ (Argentina) and $\beta = 0.80$, $\phi = 0.70$ (Uruguay) to replicate the average default frequency and recovery rate of 3.26% and 25.0% for Argentina and of 3.26% and 75.0% for Uruguay. Table 3 summarizes parameter values in our model and Appendix C reports our computation algorithm.

5.2 Numerical Results on Equilibrium Properties

We explain the equilibrium features of our theoretical model in the case of Uruguay. Appendix F shows those for the case of Argentina and the case when the creditors are the proposer—results are completely identical to those when the debtor is the proposer.

First, we explore the government's choice among repayment, preemptive restructuring and default/post-default restructuring in Figure 6. Asonuma and Trebesch (2016) were the first to show its choice for or against a preemptive restructuring before income realization, i.e., ex ante and combine with conventional choice for default/post-default restructuring or repayment after income realization i.e., ex post (Yue 2010). We find the identical results when the model incorporates endogenous production.

Panel i reports the ex-ante choice for or against a preemptive restructuring, given current debt level (vertical axis) and previous productivity realization (horizontal axis). The government opts a preemptive restructuring when debt is high and lagged productivity level is low (the blue region). When default is highly anticipated, it finds a default/post-default restructuring with high probability and larger costs—longer financial exclusion and larger productivity losses—more costly and avoids by taking a preemptive option. On the contrary, it opts a non-preemptive option when debt is low and lagged productivity level is high (the green region). When default is less anticipated, it finds certain costs associated with a preemptive restructuring option—shorter financial exclusion, smaller productivity losses, and high recovered debt payments—more costly and avoids by passing a preemptive option.

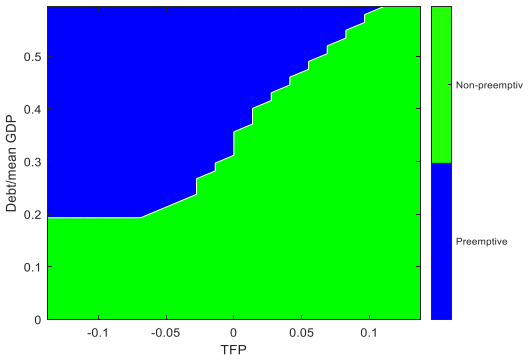
Panel ii shows the ex post choice between repayment and default/post-default restructuring, given current debt level (vertical axis) and current productivity realization (horizontal axis). As in previous work on post-default restructurings (e.g., Yue 2010), the government opts default/post-default restructuring when debt is high and current productivity level is low (the red region). It finds costs of full repayment more costly and avoids by taking a default/post-default restructuring option with costs of financial exclusion and productivity losses.

Panel iii combines the choice for or against a preemptive restructuring and that for repayment and default together given mean productivity realization in the previous period. The government's preemptive restructuring choice (the blue region) is made before current productivity realization, while its repayment and default/post-restructuring choice is made after current productivity realization. Before current productivity realization, i.e., ex ante, the sovereign takes a preemptive restructuring approach when default is highly anticipated—high debt and low productivity. Otherwise, after current productivity realization, i.e., ex post, the government takes a default/post-default restructuring choice when current productivity is low, while repays debt

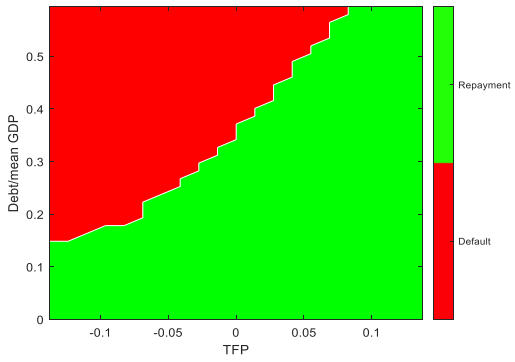
Figure 6: Debtor's Equilibrium Choice (Uruguay)

Part A: Public Capital at the Mean Level

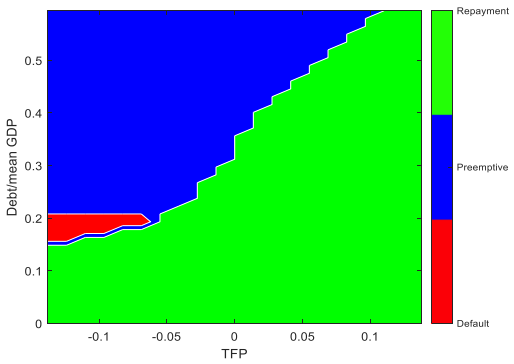
(i) Choice for Preemptive Restructuring



(ii) Choice for Default/Post-default Restructuring



(iii) Choice for Repayment, Preemptive and Default/Post-default Restructuring



in full when current productivity is high.

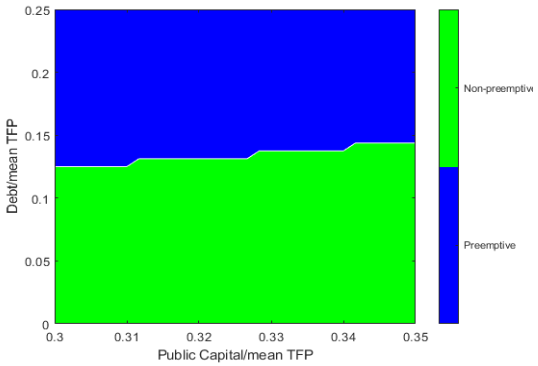
Second, we explore the government's expenditure consolidation choice in Figure 7. We classify three types of expenditure consolidation: (i) *hard* (the black region) as a change in public expenditure-to-mean GDP ratio is more than 10 percent below the average public expenditure-to-mean GDP ratio, (ii) *soft* (the purple region) as that from 10 percent below the average public expenditure-to-mean GDP ratio to zero (i.e., constant public expenditure-to-mean GDP ratio), (iii) *no* (the yellow region) otherwise.[16] Our main novelties are two-holds: soft expenditure consolidation in an intermediate credit record and hard expenditure consolidation in a good credit record. These are new contributions to the literature which only explains hard expenditure consolidation in a bad credit record (i.e., default or post-default restructuring, Cuadra et al.

[16]In the literature of sovereign debt and fiscal policy, hard expenditure consolidation in a bad credit record still exists as an optimal choice given specific states. In simulation exercise based on reasonably calibrated shock process and optimal government's response, however, these states do not materialize and as a result, the choice is not selected as an equilibrium path.

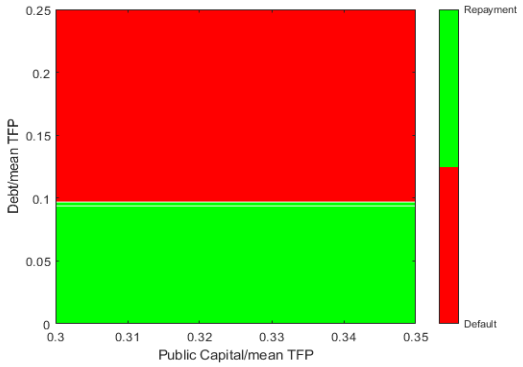
Figure 6: Debtor's Equilibrium Choice (Uruguay) (cont.)

Part B: Mean lagged TFP and low current TFP

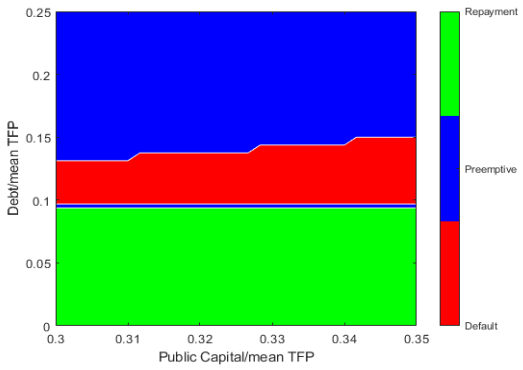
(i) Choice for Preemptive Restructuring



(ii) Choice for Default/Post-default Restructuring



(iii) Choice for Repayment, Preemptive and Default/Post-default Restructuring



2010, Arellano and Bai 2017).

Panel i shows its choice under intermediate and bad credit records given current debt level (vertical axis) and current productivity realization (horizontal axis). When the government has a bad credit record (i.e., post-default restructuring)—corresponding to the red region in panel iii in Figure 6—, it takes hard expenditure consolidation (the black region). As in Asonuma and Joo (2022), hard expenditure consolidation is comprised of a mild reduction on public consumption and transfers and a large decline in public investment. Key drivers of these factors are the government's consumption smoothing motive, larger productivity losses, and tight fiscal constraint.

When it has an intermediate credit record (i.e., preemptive restructuring)—corresponding to the blue region in panel iii in Figure 6—, it takes soft expenditure consolidation (the purple region). Soft expenditure consolidation is comprised of a smaller reduction in public consumption and transfers and a milder decline in public investment. The reduction in public consumption and transfers is smaller than that in hard expenditure consolidation and the decline in public

investment is milder than that in hard expenditure consolidation because because of smaller productivity losses in preemptive restructurings following ex ante expenditure consolidation prior to restructurings.

Panel ii shows its choice under a good credit record. When default is highly anticipated given high debt and low current productivity, the government takes hard expenditure consolidation (the black region). Hard expenditure consolidation is comprised of a mild reduction on public consumption and transfers and a large decline in public investment. A main driver of these two factors is high debt payments, different from the aforementioned drivers in a bad credit record. The government with high debt payments is forced to take hard expenditure consolidation given high anticipated default risk. When implementing hard consolidation, the government with consumption-smoothing motive has no choice but to reduce public investment sharply, which in turn, results in low level of public capital. With high effective costs of a default and post-default restructuring, the government opts to take a preemptive restructuring in subsequent periods.

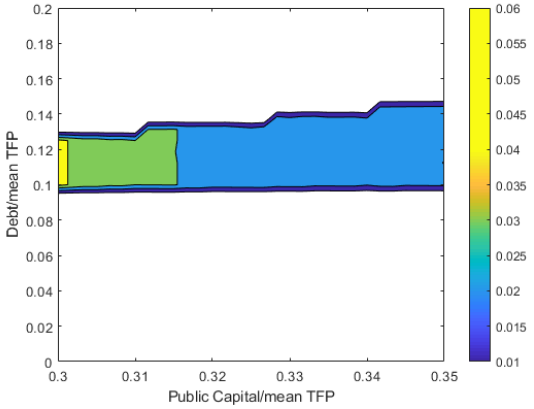
On the contrary, when a default is not anticipated given moderate level of debt and moderate level of productivity, it takes no expenditure consolidation (the yellow region). In this case, government with moderate debt payments is not forced to take expenditure consolidation, given low anticipated default risk. Given low effective costs of a default and post-default restructuring due to high level of public capital—after no sharp reduction in public investment—, the government is willing to gamble. Upon realization of severe productive shocks, the government is forced to take a default/post-default restructuring choice and implement hard expenditure consolidation.

In panel iii, we combine panels i and ii in Figure 7 with the choice of repayment, preemptive and default/post-default in panel iii in Figure 6. When the government takes a preemptive restructuring option prior to current productivity realization, it takes soft expenditure consolidation (the blue region), corresponding expenditure consolidation during a preemptive restructuring. After it passes a preemptive option, there are four cases. When it defaults after low productivity realization, it takes hard expenditure consolidation (the red region), corresponding to "ex post expenditure consolidation" upon a default and post-default restructuring. When it repays debt in full after low productivity realization, it takes hard expenditure consolidation (the green region), corresponding to "ex ante expenditure consolidation" prior to a preemptive restructuring. By taking hard expenditure consolidation, it could avoid an anticipated future default with large costs.

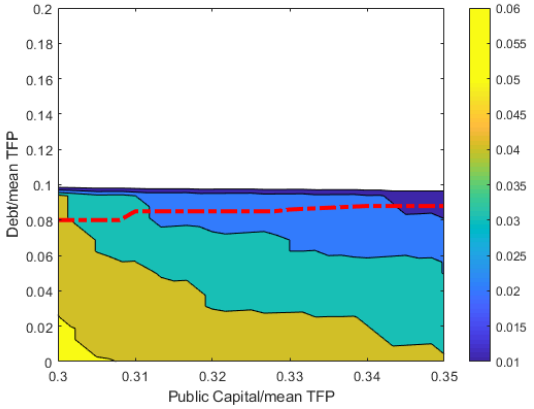
Third, Figure 8 reports the government's choice between settlement and delay conditional on strategies of expenditure consolidation and debt restructuring in the previous period. What differentiates our model from previous work on delays in renegotiations (Benjamin and Wright 2013, Bi 2008; Asonuma and Joo 2020, 2022) is how expenditure consolidation influences the choice of settlement and delay. In the case of ex ante expenditure consolidation and preemptive restructuring, the government is more willing to settle when current productivity is at or above its mean level (panel ii). Due to smaller productivity losses and "soft" expenditure consolidation in the previous period, it has more resources to allocate on debt settlement (i.e., loose fiscal constraint) and public capital accumulates quickly. The "soft" expenditure consolidation further

Figure 7: Debtor's Investment Choice (Uruguay)

(i) Under Default Choice (low current TFP)



(ii) Under Repayment Choice (low current TFP)



facilitates quicker capital accumulation in addition to smaller productivity losses which are considered to be a main driver for quick settlement in preemptive restructurings (Asonuma and Trebesch 2016). In the case of ex post expenditure consolidation and post-default restructuring, the government is more willing to delay when debt is high or moderate and current productivity is low or moderate (panel i). Two drivers of delays are recovery in productivity (Benjamin and Wright 2013; Bi 2008) and slow public capital accumulation (Asonuma and Joo 2022). Given larger productivity losses and hard expenditure consolidation in the previous period, public capital accumulation is slow resulting in slow recovery in production (i.e., repayment capacity).

5.3 Simulation Exercise

We apply a simulation exercise and discuss how our model successfully replicates moments in Argentine default and post-default restructuring in 2001–05 and Uruguayan preemptive restructuring in 2003. As in previous quantitative work on sovereign default, we conduct 1000 rounds of simulation, pull out 40 observations prior to and observations during post-default and preemptive restructuring episodes.

For private sector, GDP, consumption and the net exports (percent of GDP) are in a quarterly frequency and seasonally adjusted. Argentine dataset is from the MECON and covers (i) 1993Q1–2001Q4 (pre-restructuring period) and (ii) 2002Q1–2005Q2 (restructuring period), and Uruguayan dataset is from the CBU and covers (ii) 1993Q1–2002Q4 (pre-restructuring period) and (ii) 2003Q1–Q2 (restructuring period).

For public sector, expenditure components are at a yearly frequency and from Asonuma and Joo (2022). Argentine public debt data (percent of GDP) are from the MECON and cover (i) 1993–2001 (pre-restructuring period) and (ii) 2002–2005 (restructuring period). Uruguayan public debt data (percent of GDP) are from the IMF WEO and cover (i) 1993–2002 (pre-restructuring period) and (ii) 2003 (restructuring period). We use bond spreads from J.P Morgan's Emerging Markets Bond Index Global (EMBIG) for both countries: 1997Q1–2001Q4 for Argentina and 1997Q1–2002Q4 for Uruguay.

Table 4 shows both business for public sector and non-business cycle statistics in two types of restructurings. The theoretical model replicates that public consumption and transfers are procyclical and volatile as in Cuadra et al. (2010) for Mexican case.

Our stimulation results report three new findings. First, we replicate types of expenditure consolidation as observed in the data: ex ante for Uruguayan preemptive case and ex post for Argentine post-default case.

Second, our model generates lower average investment-to-GDP ratio during restructurings than in pre-restructuring periods for preemptive restructurings. Previous studies (Asonuma and Joo 2022) explain the trend only in post-default restructurings. We show a similar pattern in preemptive restructurings but driven by two different factors: a sharp reduction in public investment prior to a preemptive restructuring and quick settlement before recovery in public investment.

Third, we show that debt-to-GDP ratio is higher in restructuring period than that in pre-restructuring period in both type of restructurings. Moreover, debt-to-GDP ratio matches well with the data in both restructuring and pre-restructuring periods for both types of restructurings. These are driven by both multi-round renegotiations for both types of restructurings, different from one-round negotiation (Asonuma and Trebesch 2016).

Figures 8 contrasts data and simulation results on the strategies of expenditure consolidation and debt restructurings, dynamics of public investment and consumption and transfers, recoveries in public investment and restructuring duration. Panels (i)-(iv) in Figure 8 follow the same presentation format as in Figures 1-4. Solid lines in blue show the Argentine and Uruguay data, dash lines in red show our baseline model.

Table 4: Simulation Results of Models

(i) Business Cycle Statistics

	Uruguay 2003		Argentina 2001-2005	
	Data	Baseline Model	Data	Baseline Model
Target statistics				
Pre-restructuring period				
Average public consumption & transfers/GDP ratio (%)	19.4	20.5	20.0	22.9
Public investment (std. dev.)/output (std. dev.)	5.8	3.04	5.1	5.9
Restructuring period				
Average output deviation during debt renegotiations (%)	-2.28	-3.0	-3.47	-4.50
Non-target statistics				
Pre-restructuring period				
Public sector				
Public consumption & transfers (std. dev.)/output (std. dev.)	1.09	1.00	1.26	1.23
Corr.(public consumption & transfers, output)	0.35	0.74	0.52	0.85
Average public investment/GDP ratio (%)	4.18	3.70	1.31	1.60
Average public expenditure/GDP ratio (%)	23.5	24.2	21.3	23.5
Average public investment/public expenditure ratio (%)	16.9	14.7	6.2	6.4
Restructuring period				
Public sector				
Public consumption & transfers (std. dev.)/output (std. dev.)	2.0[1/]	0.78	0.99	2.36
Corr.(public consumption & transfers, output)	1.0[1/]	0.89	0.99	0.77
Average public consumption & transfers/GDP ratio (%)	25.2	20.7	20.2	23.3
Average public investment/GDP ratio (%)	3.20	3.25	1.19	1.47
Average public expenditure/GDP ratio (%)	28.4	23.9	21.3	24.7
Average public investment/public expenditure ratio (%)	11.2	15.8	5.7	5.9
Expenditure consolidation choice	ex ante	ex ante	ex post	ex post

(ii) Non-business Cycle Statistics

	Uruguay 2003		Argentina 2001-2005	
	Data	Baseline Model	Data	Baseline Model
Target statistics				
Default probability (%)	3.26	3.03	3.26	3.05
Average recovery rate (%)	87.1	83.0	25.0	27.1
Pre-restructuring period				
Average debt/GDP ratio (%)	59.1	48.0	45.4	44.7
Bond spreads: average (%)	7.7	1.03	9.4	1.65
Bond spreads: std. dev. (%)	5.1	1.50	7.6	2.25
Corr.(debt/GDP, spreads)	1.00	0.11	0.92	0.18
Restructuring period				
Restructuring strategy	preemptive	preemptive	post-default	post-default
Average debt/GDP ratio (%)	130.5	51.6	130.5	50.7
Duration of renegotiations/ exclusion (quarters)	1.0	4.3	14.0	11.2
Average public investment recovery (quarterly) from t-1 to pre-restructuring level	10.3	7.5	12.0	8.5

Notes: [1/] Statistic is based on 2003-04 in Uruguay.

First and importantly, panel (i) in Figure 8 shows that our model replicates three dominant strategies of expenditure consolidation and restructurings as in the data. Ex post expenditure consolidation accounts for 60 percent of post-default restructurings, while ex ante expenditure consolidation accounts for 58 percent of preemptive restructurings and 57 percent of non-restructuring debt distress, respectively. Moreover, when our results remain robust when we use an alternative definition of expenditure consolidation based on expenditure-to-lagged GDP ratio (Figure D1 in Appendix D.).

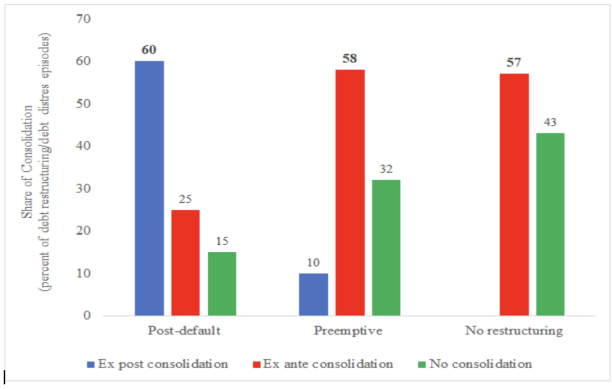
Second, panel (ii) in Figure 8 shows that in our model, public investment declines sharply at the start of post-default restructurings and recovers gradually. In preemptive restructurings, public investment declines sharply prior to the start of restructurings and recovers partially. On the contrary, in non-restructuring debt distress, public investment public investment decline temporally and recovery quickly. All these public investment dynamics are in line with the data.

Third, panel (i) in Figure 8 shows that as in the data, recoveries in public investment are shorter in preemptive restructurings than in post-default restructurings (1.8 years vs. 2.0 years).

Fourthly, panel (ii) in Figure 8 shows that in both post-default and preemptive restructurings, public consumption and transfers decline temporarily at the start of restructurings and recover quickly. In non-restructuring debt distress, public consumption and transfers are on a steady upward trend.

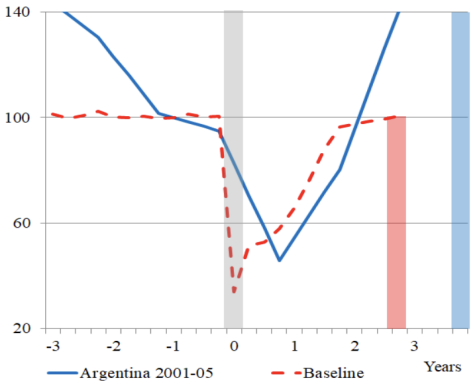
Figure 8: Simulation Results on Argentina and Uruguay

(i) Strategies of Fiscal Expenditure Consolidation
Debt Restructurings

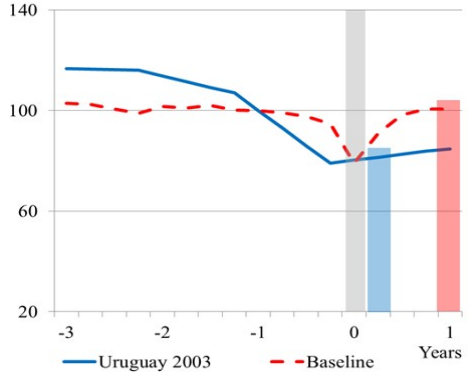


(ii) Public Investment around Debt Restructurings and Debt Distress

(a) Post-default Restructuring (Argentina)



(b) Preemptive Restructuring (Uruguay)



(c) Non-restructuring Debt Distress (Argentina)

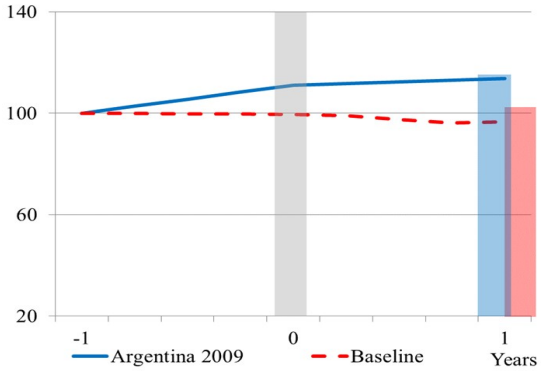
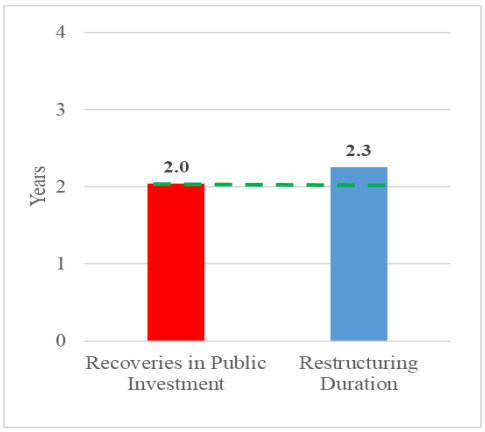


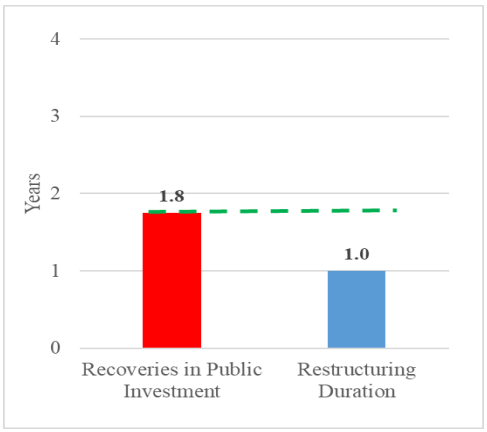
Figure 8: Simulation Results on Argentina and Uruguay (cont.)

(iii) Recoveries in Public Investment and Restructuring Duration

(a) Post-default Restructuring (Argentina)

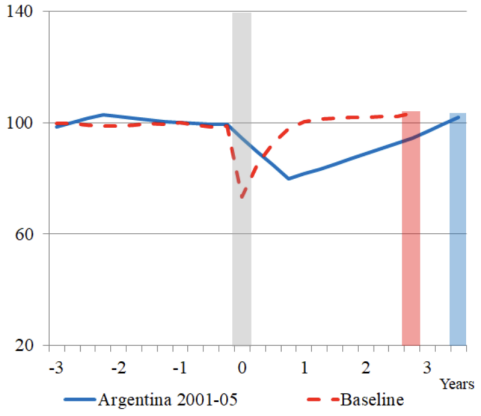


(b) Preemptive Restructuring (Uruguay)

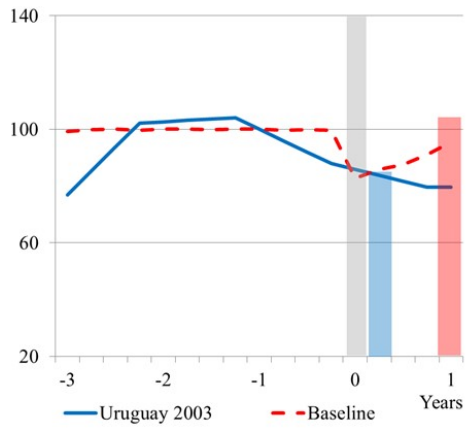


(iv) Public Consumption and Transfers around Restructurings and Debt Distress

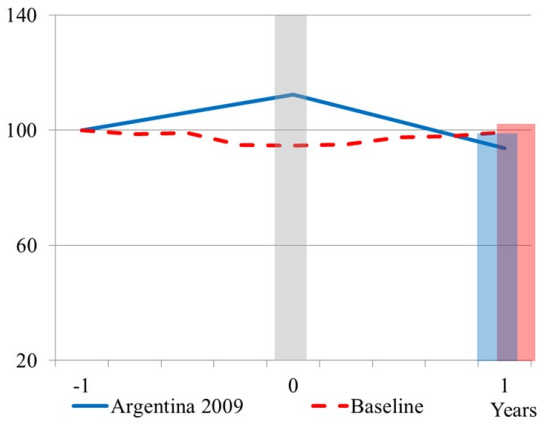
(a) Post-default Restructuring (Argentina)



(b) Preemptive Restructuring (Uruguay)



(c) Non-restructuring Debt Distress (Argentina)

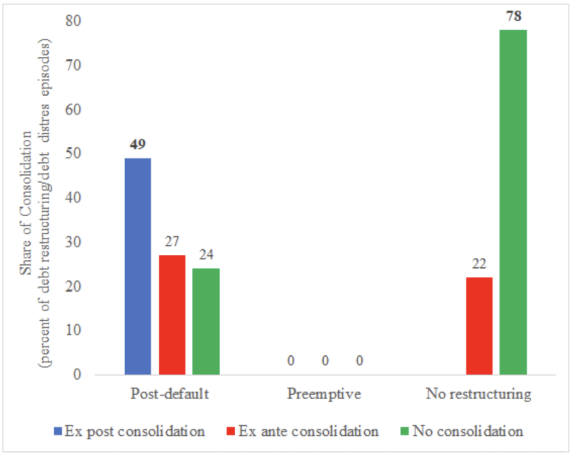


5.4 Roles of Preemptive Restructuring Choice and Public Capital

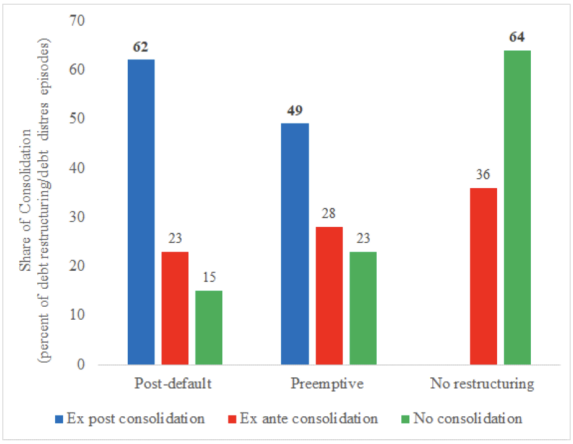
In our model, the government's choice of two types of restructurings and public capital accumulation are two main drivers to account for both ex ante expenditure consolidation and preemptive restructuring, and ex ante expenditure consolidation and no restructuring. To shed light on two drivers, we contrast our model with two different models.

Figure 9: Strategies of Expenditure Consolidation and Debt Restructurings

(i) No Preemptive Restructuring Choice



(ii) Fixed Public Capital



Preemptive Restructuring Choice

Panel (i) in Figure 9 shows the government's strategies in the model without a preemptive restructuring option. Ex post expenditure consolidation continues to be a dominant strategy in post-default restructurings (left section). No restructuring choice becomes less frequent and conditional on the choice, no expenditure consolidation becomes a dominant strategy (right section). When the government does not have preemptive restructuring choice, an advantage associated with ex ante expenditure consolidation is reduced since a preemptive restructuring does not follow the ex ante expenditure consolidation. As a result, the government is less willing to take a choice of ex ante expenditure consolidation and no restructuring.

Public Capital

Panel (ii) in Figure 9 shows the government's strategies in the model with fixed public capital. Ex post expenditure consolidation remains as a dominant strategy in post-default restructurings (left section). On the contrary, ex post expenditure consolidation turns out to be a dominant strategy in preemptive restructurings (middle section). Moreover, no expenditure consolidation also becomes a dominant strategy in non-debt restructuring debt distress (right section). When capital remains at fixed level, ex ante expenditure consolidation urges the government to reduce

sharply public consumption and transfers. The government with consumption-smoothing motive is less willing to take the ex ante expenditure consolidation.

6 Conclusion

This paper newly sheds light on the sovereign governments' strategies of fiscal expenditure consolidation and debt restructurings on sovereign debt crises and resolution. We compile a novel dataset on strategies of fiscal expenditure consolidation and debt restructurings in 1975–2020. We find new five stylized facts on expenditure consolidation and debt restructurings. To explain these stylized facts, we build a theoretical sovereign debt model that embeds endogenous choice of preemptive and post-default renegotiations, public capital accumulation, and expenditure composition. The model quantitatively shows the sovereign's choice of ex ante expenditure consolidation and preemptive restructuring results in quick debt settlement and public investment recovery. Data support our theoretical predictions.

Current research explores empirically multiplier effects of "planned (announced)" fiscal consolidation during recessions and crises (e.g., Alesina et al. 2019; Auerbach and Gorodnichenko 2012). A potential research question is how "planned" fiscal consolidation—the government's choice independent from business cycles and shocks—differs from "endogenous" fiscal consolidation—the government's optimal response to business cycles and shocks. For future work, we can explore whether the government's fiscal consolidation choice is dependent or independent on business cycles and external shocks might result in different outcomes as observed in the data.

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Appendix A Datasets

A.1 Non-restructuring Debt Distress

Table A1: Non-restructuring Debt Distress Episodes

Country	**Non-restructuring Debt Distress Periods**		**Estimated Probability** Post-default	Preemptive	**EMBIG Bond Spreads**	**No Overlap with Restructurings**	**Being Cured** (an interval of 2 years)
	start	end	(percent)	(percent)	(basis points)	(Yes/No)	(yes/No)
Algeria	1982	1982	15.1			Yes	Yes
Algeria	1984	1984	15.0			Yes	Yes
Argentina	1975	1975	25.5			Yes	Yes
Argentina	1995	1995			1,232	Yes	Yes
Argentina	2009	2009			1,174	Yes	Yes
Brazil	1995	1995			1,108	Yes	Yes
Brazil	2002	2002			1,418	Yes	Yes
Congo, Rep.	1976	1976	19.6			Yes	Yes
Costa Rica	1991	1991	17.0			Yes	Yes
Ecuador	2001	2002	18.0			Yes	Yes
Jordan	1982	1982	14.8			Yes	Yes
Kenya	1982	1982	19.9			Yes	Yes
Madagascar	1975	1975	14.8			Yes	Yes
Malawi	1984	1984	30.0			Yes	Yes
Mauritania	1982	1982	29.5			Yes	Yes
Mauritania	1984	1985	17.9			Yes	Yes
Mexico	1985	1985			1,142	Yes	Yes
Mongolia	2010	2010		39.9		Yes	Yes
Nigeria	1992	1995	34.5		1,910	Yes	Yes
Nigeria	2001	2001	15.9			Yes	Yes
Pakistan	2008	2009			1,118	Yes	Yes
Seychelles	1982	1982	15.4			Yes	Yes
Togo	1983	1984	34.6			Yes	Yes
Ukraine	2010	2010		37.6		Yes	Yes
Venezuela, RB	2009	2011			1,191	Yes	Yes

A.2 Fiscal Expenditure Consolidation Episodes in 1975-2020

Table A2: Fiscal Expenditure Consolidation Episodes in 1975–2020

Restructuring index	**Country**	**Debt Crisis (Restructuring)**		**Non-restructuring Debt Distress**				
Asonuma and Trebesch (2016)		Start	Yes/No	Type	Start	Start year	End	End year
1	Albania	1981	Yes	Ex Post	T+1	1992	T+3	1994
2	Algeria	1990	Yes	Ex Ante	T-1	1989	T	1990
3	Algeria	1993	Yes	Ex Post	T+1	1994	T+2	1995
4	Argentina	1982	No					
5	Argentina	1985	No					
6	Argentina	1988	No					
7	Argentina	2001	No					
8	Argentina	2019	No					
9	Barbados	2018	No					
10	Belize	2006	Yes	Ex Ante	T-2	2004	T-1	2005
11	Belize	2012	No					
12	Belize	2016	No					
13	Belize	2020	No					
14	Bolivia	1980	Yes	Ex Post	T+5	1985	T+6	1986
15	Bolivia	1988	Yes	Ex Ante	T-2	1986	T	1988
16	Bosnia and Herzegovina	1992	Yes	Ex Post	T+5	1997	T+5	1997
17	Brazil	1982	No					
18	Brazil	1983	No					
19	Brazil	1984	No					
20	Brazil	1986	No					
21	Brazil	1989	No					
22	Brazil	1989	No					
23	Bulgaria	1990	Yes	Ex Post	T+4	1994	T+4	1994
24	Cameroon	1985	Yes	Ex Post	T+9	1994	T+9	1994
25	Chad	2014	No					
26	Chad	2017	No					
27	Chile	1983	Yes	Ex Post	T	1983	T	1983
28	Chile	1983	Yes	Ex Post	T	1983	T	1983
29	Chile	1984	Yes	Ex Ante	T-1	1983	T-1	1983
30	Chile	1986	Yes	Ex Ante	T-1	1984	T-1	1985

A.3 Joint Strategies of Fiscal Expenditure Consolidation and Debt Restructurings

There are eight joint strategies of fiscal expenditure consolidation and debt restructurings:

- Default/Post-default restructuring
  - Ex post fiscal expenditure consolidation
  - Ex ante fiscal expenditure consolidation
  - No fiscal expenditure consolidation
- Preemptive debt restructuring
  - Ex post fiscal expenditure consolidation
  - Ex ante fiscal expenditure consolidation
  - No fiscal expenditure consolidation
- Non-restructuring debt distress
  - Ex ante fiscal expenditure consolidation
  - No fiscal expenditure consolidation

A joint strategy of ex post fiscal consolidation and non-restructuring debt distress does not exist because no debt crisis occurs.

There are two joint strategies of fiscal consolidation and non-debt crisis recessions:

- Non-debt crisis recession
  - Ex ante fiscal expenditure consolidation
  - No fiscal expenditure consolidation

A joint strategy of ex post fiscal consolidation and non-debt crisis recession does not exist because no debt crisis occurs.

Appendix B Further Empirical Analysis

Figure B1: Strategies of Fiscal Expenditure Consolidation and Debt Restructurings in 1975–2020—Alternative Classification

(i) Expenditure-to-lagged GDP ratio

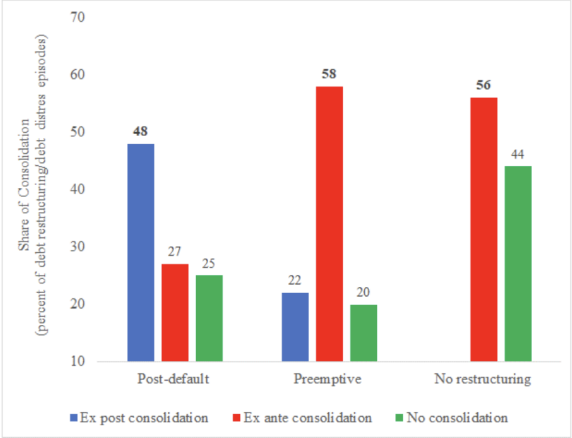


Figure B2: Strategies of Fiscal Expenditure Consolidation around Non-debt Crisis Recessions in 1975–2020

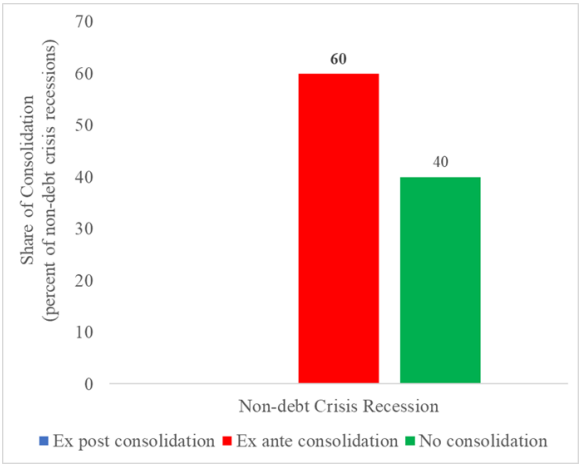
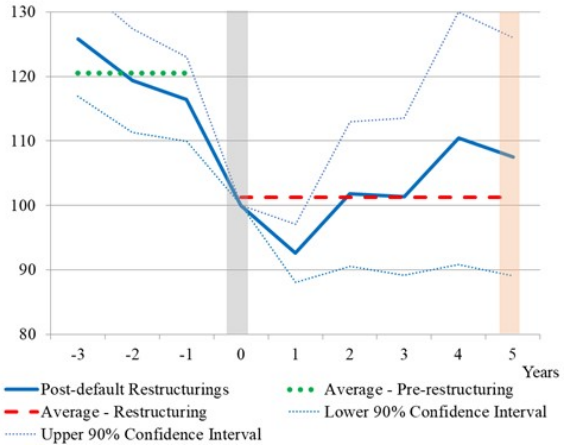
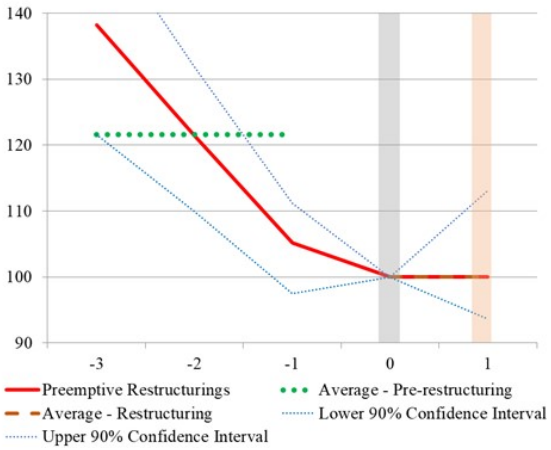


Figure B3: Public Investment (percent of GDP)

(i) Post-default Restructurings



(ii) Preemptive Restructurings



(iii) Non-restructuring Debt Distress

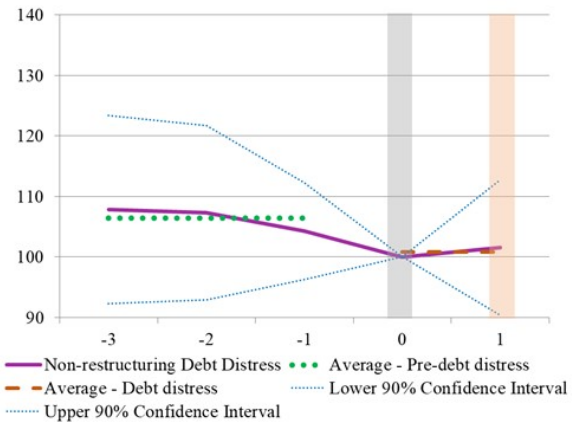


Figure B4: Public Investment (Level)

(i) Non-debt Crisis Recessions

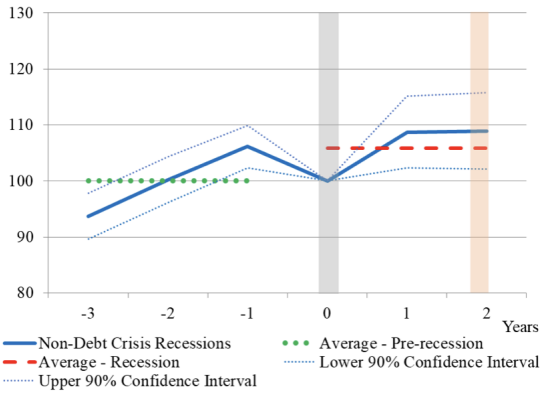
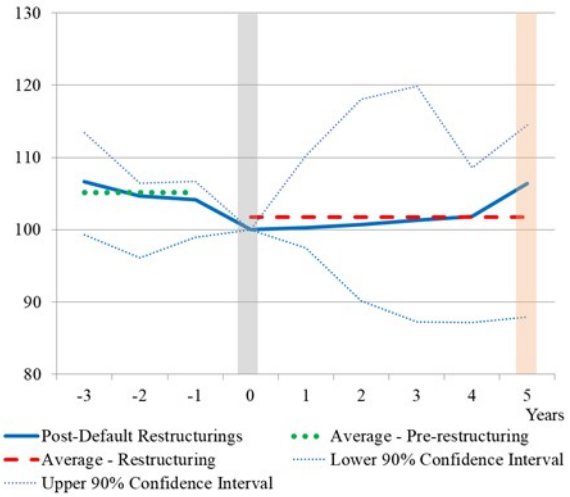
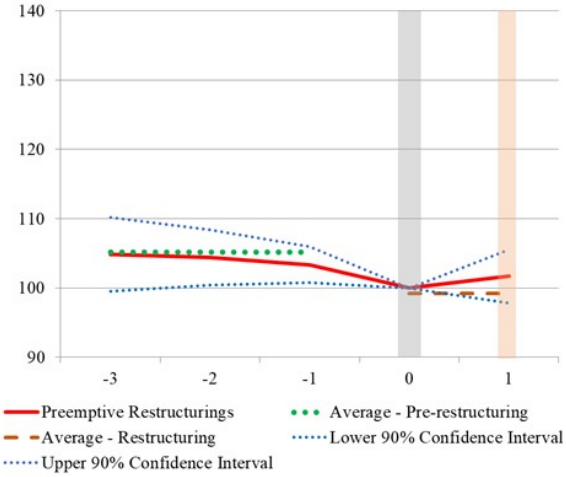


Figure B5: Public Consumption and Transfers (percent of GDP)

(i) Post-default Restructurings



(ii) Preemptive Restructurings



(iii) Non-restructuring Debt Distress

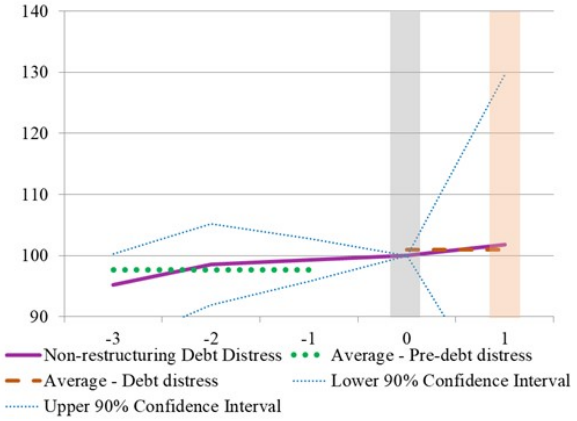


Figure B6: Public Consumption and Transfers (Level)

(i) Non-debt Crisis Recessions

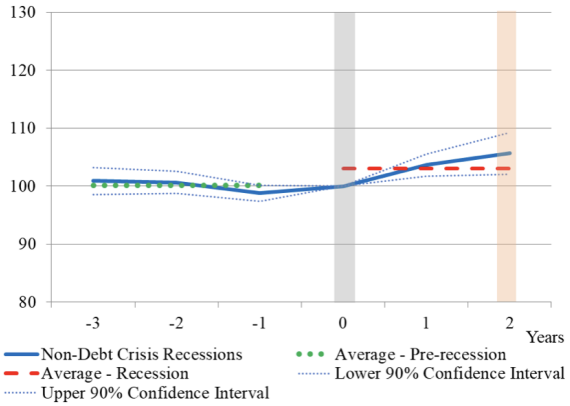
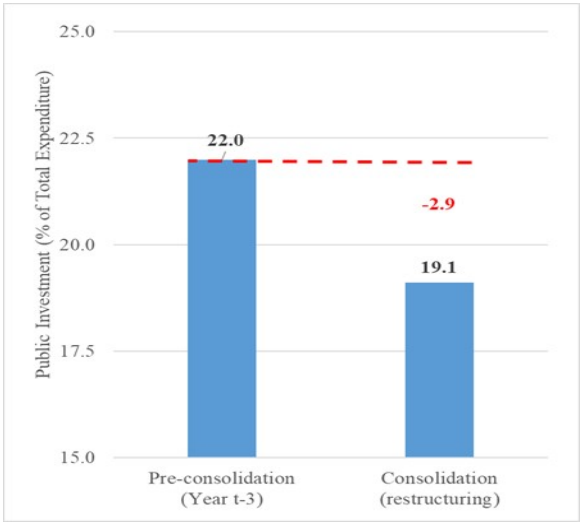
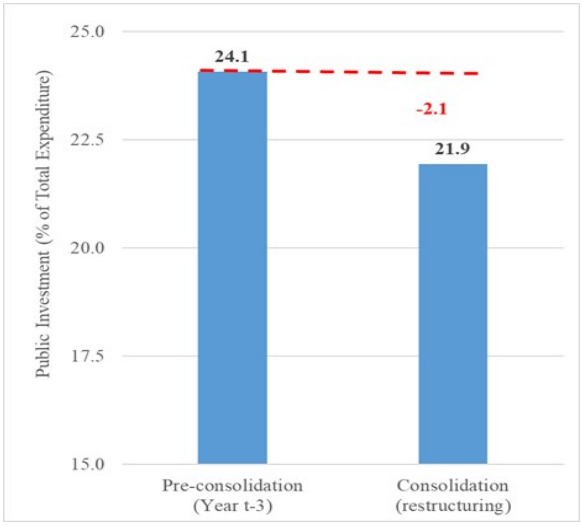


Figure B7: Public Expenditure Composition

(i) Post-default Restructuring



(ii) Preemptive Restructuring



Appendix D

Figure D1: Strategies of Fiscal Expenditure Consolidation and Debt Restructurings —Alternative Definitions

(i) Expenditure-to-lagged GDP ratio

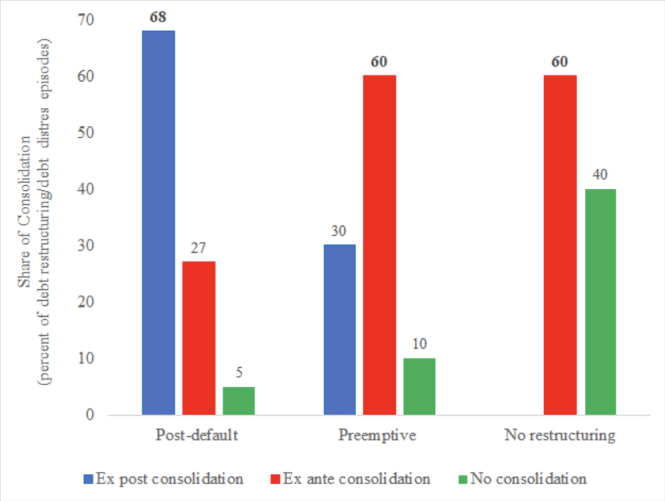
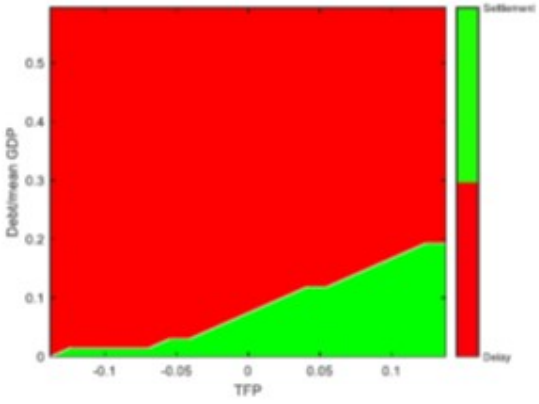


Figure D2: Debtor's Equilibrium Choice of Settlement and Delay (Uruguay)

(i) Ex post and Post-default



(ii) Ex ante and Preemptive