Attenuated or Augmented? Monetary Policy Actions under Uncertain Economic State and Learning

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Abstract

Monetary policy is conducted in an environment of considerable uncertainty. In particular, Bernanke (2007) emphasizes that monetary authority faces substantial challenges in determining the sources of variation in macroeconomic variables. What can policy-makers do when supply and demand disturbances are unobserved and indistinguishable? Is a policy rule widely conducted otherwise still appropriate? I explore these questions by developing a novel New Keynesian DSGE model featuring uncertain economic state characterized by the unobservability and indifferentiability between supply-side total factor productivity (TFP) shock and demand-side investment specific technology (IST) shock. Private agents and policy-makers have access to noisy signals about these two productivities, learn from those signals, and make decisions based on subjective expectations. Given the estimated model, I show that, compared to the environment without uncertainty, economic agents behave similarly if shocks can be observed or quickly recognized, but differently if shocks take time to be revealed. Furthermore, a policy following the Taylor rule, which responds to inflation and output gap, still has the power to accommodate the TFP shock and offset the IST shock, even though these shocks are not observed or differentiable. Finally, the uncertainty generates different welfare implications for monetary policies: i) the optimal policy, despite minimizing the aggregate volatility and thus welfare losses, can achieve only a constrained efficient state associated with an inefficiently low output level, and ii) Compared to the Taylor rule, the inflation targeting rule generates higher welfare under uncertainty, but lower welfare under certainty. This implies that policy actions should be adjusted in accordance with different economic environments.

Keywords: monetary policy, uncertainty, information friction, investment specific technology, total factor productivity, Bayesian learning

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"The Fed faces many uncertainties, and must adjust its one policy instrument to navigate as best it can this sea of uncertainty..."

———William Poole (1998)

"Uncertainty–about the state of the economy...–is a pervasive feature of monetary policy making...Notably, we now appreciate that policy decisions under uncertainty must take into account a range of possible scenarios about the state or structure of the economy, and those policy decisions may look quite different from those that would be optimal under certainty"

———Ben Bernanke (2007)

1 Introduction

Monetary stabilization policy conducted in terms of the interest rate or other implements has been extensively studied in the literature and at central banks. The most commonly used New Keynesian framework to evaluate policy actions is based on the assumption that economic agents can observe the true state of the economy, except for some unforecastable future random shocks. If the central bank could differentiate various shocks hitting the economy and make policies accordingly, the policy action would be simple: accommodating supply shocks and offsetting demand shocks. However, as emphasized by Bernanke (2007), policy-makers face substantial challenges in determining the sources of variation in macroeconomic variables. What can policy-makers do when supply and demand disturbances are unobserved and indistinguishable? In particular, is a widely conducted policy rule still appropriate to stabilize the economy? What constraints does it create for the optimal policy making, if any? What are the welfare effects of different policy actions in this case?

I address these questions by developing a novel New Keynesian DSGE model featuring uncertain economic state and investigating its implications on economic dynamics and monetary policy. This framework allows for a micro-founded treatment of uncertainty on supply and demand disturbances, specified as the unobservability and indistinguishability of the supply-side total factor productivity (TFP) shock and the demand-side investment specific technology (IST) shock[1].[2] In the modeled economy, private agents and policy-makers, who are subject to information friction, can never separately observe or distinguish the TFP and IST shocks. Instead, they know the processes disciplining the evolution of these unobserved productivities and have access to two signals of them. One signal is the realized output, which provides combined information about TFP and the effective capital that is dependent on IST. The other

[1] TFP measures how effectively inputs are utilized in production. IST is associated with improvement in the quality of investment goods that becomes embodied in the efficient capital. The US tech boom-bust cycle around the new millennium illustrates the possibility of the indistinguishability between IST and TFP as a source of economic fluctuations. See details in Appendix 1.

[2] As discussed in Smets and Wouters (2007) and Justiniano et al. (2010), the TFP and IST shocks are representatives of supply and demand shocks respectively, in the sense that an increase of TFP will decrease inflation, whereas that of IST will increase inflation.

signal is a noisy indicator of IST. The error term in this signal is interpreted as "sentiment" shock, which is unwarranted by the underlying fundamentals but could alter public expectations about IST. The information friction partially decouples agents' choices from the actual fundamentals but connects them with signals and subjective beliefs. In particular, labor and capital markets have to clear based on pre-agreed factor payment contracts rather than on the ex post marginal contributions to production; firms' pricing decisions depend on expected marginal costs, which yields an information-constrained Phillips curve; households choose investment in accordance with the expected but not the actual IST. Economic agents update subjective beliefs through Bayesian learning when new informative signals arrive. Before subjective beliefs align with the underlying fundamentals, there are temporary expectation errors that can cause the choices to differ from those under certainty, which will generate different economic dynamics. Policy-makers share the same information set and learning mechanism with private agents. They make and conduct policies in accordance with their (and public) expectations about the sources of variation.

In principle, it is difficult to compute this general equilibrium problem where the state variables, which are beliefs about the unobserved fundamentals, are continuous functions. I adopt a tractable solution strategy similar to that developed by Ma and Samaniego (2015), which is able to overcome these computational challenges. I estimate the model to match some important moments of the US economy and the forecasts from the Survey of Professional Forecasters using Bayesian estimation methodology. I obtain direct measures of the characteristics of the unobserved fundamentals and estimates of the structural parameters. Based on the estimation, I study the dynamics of the modeled economy and the implications of information friction on monetary policy .

I obtain three sets of results. First, I show that, under uncertain economic state and learning, macroeconomic variables follow similar paths with those generated by standard New Keynesian models, if the shocks that hit the economy can be observed, such as government spending shock and monetary policy shock. Agents also behave similarly when the occurrence of unobserved shocks, such as the TFP shock, can be quickly recognized. However, if the unobserved shocks take time to be revealed, such as the IST shock and sentiment shock, the responses will be different since subjective beliefs influenced by "sentiment" can be persistently different from the actual fundamentals. For example, when the sentiment shock happens, the economy could follow a boom-and-bust path, which cannot be generated under certainty. Second, I demonstrate that the monetary authority, using a Taylor rule which responds to current inflation and output gap, has the power to accommodate the supply-side TFP shock and offset the demand-side IST and sentiment shocks, even though these shocks are not observed or distinguishable. The fact that monetary policy is still effective to stabilize the economy is due to the crucial ingredient that, on average, public expectations about the unobserved shocks are positively correlated with the actual movement of the corresponding shocks. As a result, macroeconomic variables respond to these shocks in the supposed direction as if the shocks were observed. Therefore, the policy rule reacting to inflation and output gap could yield desired effects. Finally, I characterize the optimal policy and investigate the welfare implications of different policy actions under uncertainty. I show that, compared to certainty environment, the design of the optimal policy differs in that i) the uncertain economic state restricts the optimal policy to achieve only a constrained efficient allocation as would be obtained under a flexible price and wage economy subject to information friction, and ii) the subsidy to offset the distortion from monopolistic competition depends on subjective beliefs. For the welfare implications, I demonstrate that, under uncertainty, the relative welfare gain achieved by the optimal policy in comparison to simple rules is less than that under certainty. Moreover, the flexible price inflation targeting rule generates higher welfare than the Taylor rule under uncertainty, but lower welfare under certainty. Therefore, the central bank must take into account a range of possible scenarios about the state of the economy, and policy actions should be adjusted relative to the certainty benchmark when necessary.

One main contribution of this paper is the exploration of a new driving force of economic dynamics, which has been neglected in real business cycle and New Keynesian literature. That is, the information friction on the TFP and IST shocks, or more broadly on the supply and demand disturbances, could shape business cycles. While these are the two broad factors that contribute to economic dynamics, they are often treated as differentiable driving forces. This paper is the first to structurally investigate how the unobservability and indistinguishability of these two factors could impact economic agents' behavior. The findings show quantitative evidence about this critical source and channel of economic fluctuations, which provides new insights on the discussion of related topics in the literature. More importantly, this paper proposes and estimates a novel and general framework that enables a comprehensive analysis of the quantitative implication of this uncertainty for policy actions. The simulation results shed light on a crucial concern for policy-makers that have not been fully investigated before: To what extent does the indifferentiability of aggregate supply and demand disturbances have impacts on an otherwise routine policy. Furthermore, the choice of an appropriate/optimal policy under uncertain economic state has been a chronic problem for the monetary authority, and this paper contributes to this decision-making by comparing various policy actions in terms of their effects on aggregate volatility and social welfare, and more essentially, by providing a general framework that enables the quantitative evaluation of different policies under uncertain economic state.

My research is related to various strands in the literature. First, this paper is related to the discussion of TFP and IST and their role in business cycles. Greenwood et al. (1997) was one of the first papers that emphasizes the importance of IST and differentiates it from TFP. Since then, various papers (Fisher, 2006, Justiniano et al., 2011) fueled the debate on their contribution to economic fluctuations. Recently, expectation driven business cycle literature (Beaudry and Portier, 2006, Jaimovich and Rebelo, 2006, and Lorenzoni, 2009) further explores these productivities by investigating how expectations about them could generate aggregate supply and/or demand disturbances. None of the previous studies, however, have quantitatively studied how the indifferentiability and confounding of IST and TFP could characterize business cycles and affect monetary policy. This paper fills in this gap and contributes to the literature by explicitly exploring that the information friction on the demand- and supply-side productivities may distort economic agents' decisions and that the unwarranted changes in the "sentiment" about IST could create boom-bust cycles by itself. More broadly, this paper provides a general micro-founded framework that can be applied to the study of implications of uncertain economic state on business cycles and policy effects.

Second, this paper is related to the literature on monetary policy under uncertainty. There are two strands of research in this literature. One direction focuses data uncertainty, which highlights the fact that the information (data) used by central banks to make policy decisions could be incorrect and investigates its implications on policy effects. For example, Rudebusch (2001) studies how the noise from the dataset could influence the optimal responses to output and inflation in a standard Taylor rule. Orphanides and Williams (2002) studies the optimal rules under uncertain natural rate of output and proposes that central bank's policy may not be effective if the information received from data implies an incorrect natural rate. However, these studies have not considered potentially positive reaction of economic agents facing data uncertainty and thus may generate misleading results. This paper attempts to structurally explore this important characteristic by allowing private agents and policy-makers to acknowledge the fact that the available data could only partially reflect the economic state and act as applied economists and econometricians, in the sense that they learn and update beliefs about the uncertain economic fundamentals instead of simply viewing them as a combination of available data and ad-hoc measurement errors.

The other direction of this literature examines various specifications of information friction. These studies investigate the implications for monetary policy design and conduct assuming that there is imperfect information only in the private sector (Evans and Honkapohja, 2003, Orhanides and Williams, 2005), or only restricting central bank's behavior whereas private sector has access to either complete common information (Aoki 2003, Svensson and Woodford, 2003, 2004), or dispersed information where heterogenous agents experience idiosyncratic productivity not observable by other agents (Angeletos and La'O, 2008, Lorenzoni, 2010). A distinctive feature of my framework is that, the information friction originates from the unobservability and indistinguishability between supply and demand disturbances, and the friction is common to both private agents and policy-makers. Absent asymmetric information, the paper allows for a dedicated investigation on the channel that how an overwhelming aggregate uncertainty could characterize agents' behavior and policy actions. A complete analysis of the quantitative implication of this aggregate uncertainty is particularly important for the accurateness and effectiveness of policy making and conduct as different types of aggregate disturbances could generate substantially different economic dynamics and therefore require different stabilization policies.

The paper is organized as follows. Section 2 describes the model and environment. Section 3 presents the numerical analysis including solution methodology, data, estimation and business cycle properties. Section 4 studies monetary policy implications. The last section concludes and suggests direction for future research.

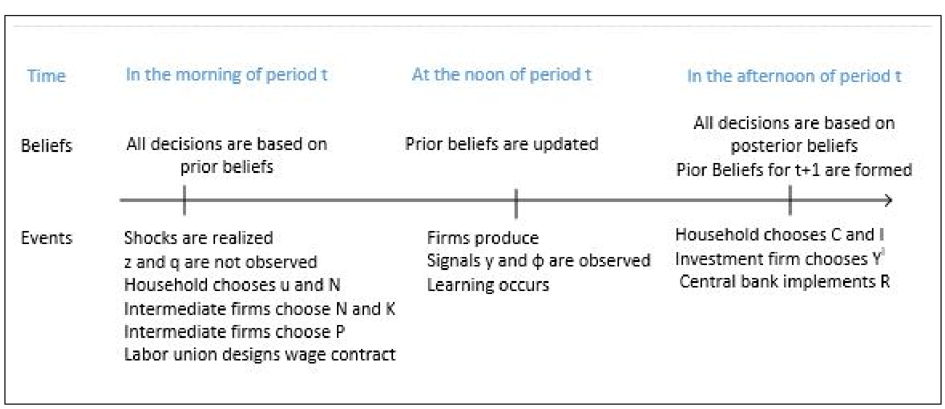
2 The model

2.1 Environment

The core of the model is a medium scale DSGE framework, based on Smets and Wouters (2007), and Justiniano et al. (2011). There are four types of agents in the decentralized economy: households, labor union, firms and monetary authority. It features habit formation, capital utilization, and investment adjustment cost in households, sticky wages in the labor market, sticky prices and fixed cost in firms, and Taylor rule and balanced budget in the monetary authority. The dynamics of the economy is driven by eight stochastic shocks, including: TFP shock, IST shock, government spending shock, wage and price mark up shocks, financial asset premium shock, and monetary policy shock. The information friction of the economy is captured by the assumption that economic agents are not able to observe TFP and IST shocks at all periods. As is shown later, this implies that the effective unit of capital stock is not observed either, for its evolution is dependent on the IST shock. Agents can observe other shocks and noisy signals regarding TFP and IST shocks, from which they obtain information about the aggregate productivities. There are two signals. One is the realized output, a combination of the realized TFP shock and effective capital stock. The other signal is a noisy indicator for the IST shock, composed of its true value and an unobserved disturbance, capturing the change in market sentiment, agents' attitude, information rigidity, and other noises that could potentially affect the perception of economic agents about the IST, which itself may lead to fluctuations in investment and output. This disturbance is the eighth shock in the model, named sentiment shock. The true data generating processes for all the shocks are exogenous and known to agents. The posterior beliefs are generated according to Bayes' rule, a combination of the subjective perceptions embodied in the prior beliefs, and the available data reflecting the stochastic processes driving the dynamics of the economy[3]. Household, labor union, all the firms and monetary authority share the same Bayesian learning process, in the sense that they observe the same signals, have the same information set and update their beliefs regarding economic fundamentals based on the same learning mechanism. For example, it is common information that the realized output depends on the realized but unobserved TFP shock, the unobserved effective capital (adjusted by utilization) and labor chosen at the beginning of the period,based on which private agents and policy-makers update their common prior beliefs following a common Bayes' rule.

[3] In this paper, I consider the case of fully Bayesian learning in the sense that economic agents take into account possible future updating of past beliefs when making current decisions.

In particular, there is a continuum of household whose objective is to maximize lifetime utility. Consumption goods and labor are two factors in the utility function. There is an exogenous habit formation in the consumption. Households own certain quantity of capital which is observable. The observed capital stock is measured, for example, in physical units, and the number of the effective units is known to be embodied in them even though that quantification is not known. The relationship between the effective capital and physical units of capital can be expressed by a non time-varying function with the feature that a higher quantity of capital is associated with a higher effective capital. The evolution of the physical unit is not a necessary information set for households' choices. Households choose the capital utilization rate, u_t, which influences the effective capital (and quantity of capital), and labor that they provide to labor union. They also choose the amount of investment given a known investment adjustment cost, but an unobserved IST shock, which influences the accumulation of effective capital in the economy. Labor union combines differentiated labor from households and supply to firms. In each period, only a fraction $1-\theta_w$ of wages can be optimally chosen by the labor union, following Calvo (1983) style. There are two types of goods that are produced in three different sectors. A continuum of intermediate goods firms produces intermediate goods, using capital and labor as inputs, which are then combined by competitive final goods producers to produce final goods. The final goods can either be consumed by households or used as inputs by investment goods producers, which produce capital and sell it to households. There are four types of prices in the economy. The price of input factors include rental rate on capital and wage on labor. The contract on the capital rent between firms and households, and on the wage between households and labor union are set prior to production, but paid after production based on the realized output. The price of final goods is a combination of intermediate goods prices. As in Calvo (1983), in each period, a fraction θ_p of intermediate goods firms cannot optimally choose their prices. The remaining fraction $1-\theta_p$ of firms choose their prices optimally to maximize the current market value of the profits while that price remains effective. There are also prices for investment goods, which reflects expectations on the investment specific technology.



Timeline

The timeline of economic agents' decisions under uncertainty is shown as above. At the beginning of each period t, stochastic shocks are realized but the IST and TFP shocks are unobserved. Before observing current signals which contain information about the current unobserved technologies, households rent capital to intermediate goods firms and provide labor to labor union. In this way, they choose capital utilization rate and labor based on the expectations about the relevant economic fundamentals. Labor union combines labor from households and provide it to intermediate goods firms according to the (expected) wage rate determined by wage contract. The wage contract is designed according to Calvo style. Intermediate goods firms use the capital and labor to maximize their expected profit under the unobserved but already realized productivity. They also choose optimal prices following Calvo style based on their beliefs on the demand and marginal costs. At the middle of the day, intermediate goods firms produce and during which, the signals are uncovered. On observing the signals, economic agents update their prior beliefs on the unobserved fundamentals to form posterior beliefs through Bayesian learning. In the afternoon of period t, final goods firms combine the intermediate goods to produce final goods. A portion of the final goods is consumed by households, and the other is used to produce investment goods and eventually the effective capital. Households make choices of investment and consumption. Monetary authority changes monetary policy if necessary. Finally, prior beliefs about the unobserved fundamentals for the next period are formed based on information obtained at this period and agents' behavior.

The details of the market participants' activities are described in the following.

2.2 Final goods sector

2.2.1 Final goods firms

Final goods firms are perfectly competitive and produce final goods by combining a continuum of intermediate goods. At each period after observing signals, they produce according to the technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1-\frac{1}{\nu_t^p}} di \right]^{\frac{\nu_t^p}{\nu_t^p - 1}}$$

where Y_t is final goods (real GDP), v_t is the elasticity of substitution between intermediate goods $Y_t(i)$. v_t is the price markup shock, a shock to the changes in the elasticity of demand and therefore in the price markup. Define $\frac{\nu_t^p}{\nu_t^p - 1} = 1 + \vartheta^p e^{\vartheta_t^p}$ where ϑ_t^p follows

$$\vartheta_t^p = \rho_p \vartheta_t^p - \varpi_p \varepsilon_{t-1}^p + \varepsilon_t^p, \varepsilon_t^p \sim N(0, \sigma_p^2)$$

Households purchase final goods at price P_t. Profit maximization and the zero profit condition for final goods firms imply the price is a CES aggregate of the prices of intermediate goods, $P_t(i)$

$$P_t = \left[\int_0^1 P_t(i)^{1-\nu_t^p} di \right]^{\frac{1}{1-\nu_t^p}}$$

where P_t is the aggregate price index. Furthermore, the demand for intermediate good i is

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\nu_t^p} Y_t$$

2.2.2 Intermediate goods firms

There is a continuum of intermediate goods firms indexed by $i \in [0, 1]$. Each firm hires labor from labor union and rents capital from households, and produces a differentiated good. The production is affected by the identical but unobserved total factor productivity as:

$$Y_t(i) = e^{z_t}(u_t k_t(i))^\alpha n_t(i)^{1-\alpha}$$

where u_t is the capital utilization rate, chosen by households, $k_t(i)$ is the effective capital stock, used by firm i in production, which is embodied in the rented physical units of capital $\hat{k}_t(i)$, $k_t(i) = f(\hat{k}_t(i))$. $f(.)$ is a non time-varying function identical to all firms, showing the relation between the effective capital and physical units of capital. $n_t(i)$ is the labor input hired from labor union. e^{z_t} is the level of total factor productivity. Again, the time series of z_t is never observed by economic agents, but the process that disciplines z_t is known to follow:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \varepsilon_t^z \sim N(0, \sigma_z^2)$$

All firms face the same isoelastic demand function given above, and take the aggregate price level P_t and final good Y_t as given. A representative firm's problem is:

$$\max_{n_t(i), \hat{k}_t(i)} E_t[P_t(i)Y_t(i) - R_t(u_t\hat{k}_t(i)) - W_t n_t(i) - \Phi] = E_t[P_t(i)e^{z_t}(u_t k_t(i))^\alpha n(i)^{1-\alpha} - R_t(u_t\hat{k}_t(i)) - W_t n_t(i) - \Phi]$$

The expectation captures the idea that, since the effective capital stock and TFP are unobserved when firms make decisions, the amount of realized output is only ex-post observed but not ex-ante known. After producing and obtaining the output, the intermediate goods firms, who price taker in the input factor market, pay at the prevailing and *pre-agreed* contract prices for capital, R_t and for labor W_t. R_t and W_t are based on the expectation about the underlying fundamentals. The contract is designed such that labor will receive its ex ante expected marginal product, and the rest of the output will be paid to capital owners. Therefore, there may be discrepancy between the pre-agreed factor payments and the ex post marginal products of these factors. Φ is a constant fixed cost, so that the profit of each firm is zero in equilibrium.

This profit maximization can also be written in terms of cost minimization problem, conditional on the output $Y_t(i)$ produced. In real terms this problem can be written as:

$$\min_{n_t(i),\hat{k}_t(i)} E_t[\frac{W_t}{P_t(i)} n_t(i) + \frac{R_t}{P_t(i)}(u_t\hat{k}_t(i)) + Z_t(i)(Y_t(i) - e^{z_t}(u_t k_t(i))^{\alpha} n_t(i)^{1-\alpha})]$$

where Z_t is the multiplier associated with the constraint. The demand for labor and capital is based on the wage and rental contracts as described above:

$$\frac{W_t}{P_t(i)} = (1-\alpha)E_t[Z_t(i)e^{z_t}(u_t k_t(i))^{\alpha} n(i)^{-\alpha}] = \frac{E[Z_t(i)Y_t(i)]}{Y_t(i)} \frac{(1-\alpha)Y_t(i)}{n_t(i)}$$

$$\frac{R_t}{P_t(i)} = \frac{E[Z_t(i)Y_t(i)]}{Y_t(i)} \frac{\alpha Y_t(i)}{u_t\hat{k}_t(i)}$$

We can write the expected real cost function (excluding fixed cost) as:

$$E_t COST_t(i) = \frac{W_t}{P_t(i)} n_t(i) + \frac{R_t}{P_t(i)}(u_t\hat{k}_t(i)) = \frac{E_t[Z_t(i)Y_t(i)]}{Z_t(i)Y_t(i)} Z_t(i)Y_t(i)$$

For this reason, $Z_t(i)$ can be thought as the standard real marginal cost, dependent on the market power of the monopoly. In equilibrium, since all firms have the same expectations and face the same maximization problems, the capital-labor ratio is the same across firms. This implies that $Z_t(i)$ will also be the same across firms, denoted as Z_t for the rest of the paper. Under information friction, however, the ex ante and ex post monopoly markup may not be the same. Since productivity and capital effectiveness are not known, the pre-determined cost depends on the expected value of the unobserved fundamentals. The expected marginal cost may be different from the realized one as there may be discrepancy between the expected output and realized output. This means the firm might charge a different price ex post compared to ex ante. This is an important feature of the economy with imperfect information, as it can influence the dynamics of inflation and the monetary authority's optimal policy choice, which will be explained in details later.

Before producing, given the expected marginal cost, intermediate goods firms also choose prices for their products. Note that in a certainty environment with flexible price, the relative price $P_t(i)/P_t$ will be a markup $\nu^p/(\nu^p - 1)$ over the real marginal cost. In a certainty environment with rigid nominal price, the relative price $P_t(i)/P_t$ is no longer a constant markup over the real marginal cost, but varying over time. In an uncertainty environment with rigid nominal price, the variation of the markup also reflects variation of expectation.

Following the structure proposed in Calvo (1983), each firm resets its price with probability $1-\theta_p$ in any given period, independent of the time elapsed since the last adjustment. Thus, each period a fraction $1-\theta_p$ of intermediate goods firms reset their prices, while the other θ_p keep their prices unchanged. As a result, the average duration of a price is given by $(1-\theta_p)^{-1}$. θ_p can therefore be interpreted as an index of price stickiness. The firms who have probability $1-\theta_p$ to reset prices at time t choose the price $P_t^*(i)$ that maximizes the expected discounted value of present and future profits. Formally, it solves the problem:

$$\max_{P_t^*(i)} E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \lambda_{t+s}}{\lambda_t} \left\{ P_t^*(i)(\Pi_{\tau=1}^s \pi_{t+\tau-1}^{\iota_p} \pi_*^{1-\iota_p}) Y_{t+s}(i) - P_t Z_{t+s} Y_{t+s}(i) \right\}$$

subject to demand constraint:

$$Y_{t+s}(i) = \left[\frac{P_t^*(i) \Pi_{\tau=1}^s \pi_{t+\tau-1}^{\iota_p} \pi_*^{1-\iota_p}}{P_{t+s}} \right]^{-\nu_t^p} Y_{t+s}$$

where λ_{t+s} is the marginal utility of income of the representative household that owns the firm, and $Y_{t+s}(i)$ denotes the output in period $t+s$ for a firm that last resets its price in period t. π_t is the gross inflation defined as $\pi_t = P_t/P_{t-1}$ and π_* is its steady state level. ι_p is the parameter controlling indexation of price on inflation.

Differentiate the above equation with respect to the relative price, $P_t(i)^*/P_t$ to obtain the first order condition for the optimal price:

$$E_t \left[Y_t(i) \left(\frac{P_t^*(i)}{P_t} - \frac{\nu_t^p Z_t}{\nu_t^p - 1} \right) + \sum_{s=1}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} Y_{t+s}(i) \left(\frac{P_t^*(i)(\Pi_{\tau=1}^s \pi_{t+\tau-1}^{\iota_p} \pi_*^{1-\iota_p})}{P_{t+s}} - \frac{\nu_t^p}{\nu_t^p - 1} \frac{P_t Z_{t+s}}{P_{t+s}} \right) \right] = 0$$

Compared with certainty benchmark with flexible prices, the following is worth noting:

1. Under purely flexible prices, $\theta_p = 0$; under perfect information, $E_t \left[Y_t(i) \left(\frac{P_t^*(i)}{P_t} - \frac{\nu_t^p}{\nu_t^p - 1} Z_t \right) \right] = Y_t(i) \left(\frac{P_t^*(i)}{P_t} - \frac{\nu_t^p}{\nu_t^p - 1} Z_t \right)$. Then the markup is $P_t^* = \frac{\nu_t^p}{\nu_t^p - 1} P_t Z_t$, which means optimal prices are a multiple $\frac{\nu_t^p}{\nu_t^p - 1}$ of the marginal cost.

2. When $\theta_p > 0$, but still under perfect information, the optimal price depends on expected future values of aggregate variables and nominal marginal costs. All the fluctuations in the markup are due to that firms are unable to reoptimize by choosing prices at each period.

3. When $\theta_p > 0$,and under information friction, the fluctuations in the markup not only results from firms being unable to adjust prices, but also unable to observe current economic fundamentals when making decisions. Therefore fluctuations are directly affected by expectation about both current and future economic states.

The average price level will be a CES aggregate of all prices in the economy:

$$P_t^{1-\nu_t^p} = \theta_p (P_{t-1} \pi_{t-1}^{\iota_p} \pi_*^{1-\iota_p})^{1-v_t^p} + (1-\theta_p)(P_t^*(i))^{1-\nu_t^p}$$

where P_{t-1} is previous price level, and $P_t^*(i)$ is the price level chosen by firms which can adjust prices at period t.

2.2.3 Phillips curve

Given the firms' pricing optimization, we can solve the optimal price in equilibrium, where all the firms that reset price choose the same price (having the same expectation and facing the same demand), hence

$$P_t^*(i) = P_t^*$$

Together with the expression regarding the marginal cost Z, we have

$$E_t\left[\sum_{s=0}^{\infty}(\theta\beta)^s\frac{\lambda_{t+s}}{\lambda_t}Y_{t+s|t}\left(\frac{P_t^*(\Pi_{\tau=1}^{s}\pi_{t+\tau-1}^{\iota_p}\pi_*^{1-\iota_p})}{P_{t+s}}-\frac{\nu_t^p}{\nu_t^p-1}\frac{P_tZ_{t+s}}{P_{t+s}}\right)\right]=0$$

Log linearizing the above equation around steady state level of prices where inflation is zero, we can obtain an expectation for inflation as:

$$\pi_t = \frac{\beta}{1+\beta\iota_p}E_t\pi_{t+1} + \frac{(1-\theta_p)(1-\theta_p\beta)}{(1+\beta\iota_p)\theta_p}E_t\tilde{Z}_t + \frac{\iota_p}{1+\beta\iota_p}\pi_{t-1} + \frac{1}{1+\beta\iota_p}\vartheta_t^p$$

where $\tilde{Z}_t$ is he log deviation of marginal cost from its steady state value $\frac{\nu^p-1}{\nu^p}$, the inverse of the steady state markup.

This is an expectation augmented and information restricted Phillips curve under uncertainty. It states that, given the expected and past inflation, as well as the markup shock, current inflation rises when the real marginal cost are expected to rise above its steady state, as firms who have the chance to reset prices will optimally choose a price above the economy's average price level in order that their markups are driven closer to the desired level. This Phillips curve is different from that under standard New Keynesian framework, in that the marginal cost which influences the inflation is not the realized cost, but the expected cost. As a result, whatever can affect expected marginal costs, such as incorrect beliefs or unobserved shocks, might generate different marginal costs compared to those implied by standard model without information friction.

2.3 Investment goods sector

There is a continuum of perfectively competitive firms which purchase final good Y_t^I to produce investment good I_t , and then sell it to households at price P_{It}. The production technology is given by

$$I_t = e^{q_t}Y_t^I$$

where e^{q_t} is IST and q_t follows:

$$q_t = \rho_q q_{t-1} + \varepsilon_t^q, \varepsilon_t^q \sim N(0, \sigma_q^2)$$

A representative firm chooses Y_t^I to maximize its profit:

$$\max_{Y_t^I} E_t[P_{It}I_t - P_tY_t^I] = E_t[P_{It}e^{q_t}Y_t^I - P_tY_t^I]$$

yielding

$$\frac{P_{It}}{P_t} = E(e^{q_t})$$

The expectation captures that the IST shock is unobserved. Therefore, when the price of investment goods P_{It} is determined in equilibrium, it reflects the subjective beliefs on the investment technology, which is influenced by signals reflecting both the actual technology and the attitude towards it in the economy. For example, if agents are optimistic about the advancement of the technology that will increase the effectiveness of capital, i.e., the expected IST is bigger than the actual value, the price of investment will be influenced by this optimism and becomes higher. This provides a potential explanation to the driving forces behind the 2000s dot-com bubble. With the approach of the new millennium, US economy became over optimistic about the new information technology which could bring economic expansion. Later, when it was gradually recognized that the actual level of the investment technology was not as high as expected, investors started to revise their investment plan and the boom ended and followed by a bust. This scenario is generally impossible to be captured by a standard DSGE model but can be illustrated by this model with information friction.

2.4 Households

There is a continuum of households. Before observing signals, a representative household chooses labor n_t and capital utilization rate u_t to maximize the expected lifetime utility. After observing the signals, it updates the learning about economic fundamentals, and chooses the expenditure on consumption and investment based on posterior beliefs. The household's maximization problem can be summarized in the following Bellman equation, with the learning process outlined in the learning section:

$$\begin{aligned} V(g_t^Z, g_t^Q, g_t^K, c_{t-1}(j)) &= E_t \left\{ \begin{array}{c} \max_{n_t(j),u_t(j),Y_t^I(j)} \frac{1}{1-\sigma_c}[c_t(j) - hc_{t-1}(j)]^{1-\sigma_c} - \psi\frac{n_t(j)^{1+\gamma}}{1+\gamma} \\ +\beta E_t V(g_{t+1}^Z, g_{t+1}^Q, g_{t+1}^K, c_t(j)) \end{array} \right\} \\ & s.t. \\ P_tc_t(j) + P_{It}Y_t^I(j) + B_{t+1}(j) &= R_tu_t(j)\hat{k}_t(j) + W_t(j)n_t(j) + e^{b_t}R_t^bB_t(j) + \Pi_t - P_ta(u_t(j))\hat{k}_t(j) + T_t \end{aligned}$$

where g_t^Z, g_t^Q, g_t^K denote respectively the prior beliefs on the TFP shock, IST shock, and effective capital stock. Since all the households share the same information set, the prior and posterior beliefs are identical across households. c_t is consumption. h captures habit formation. n_t is labor supplied to labor union. ψ captures the negative utility from the labor. γ is labor supply elasticity parameter. T_t is lump-sum tax. B_t is government bonds and R_t^b is the nominal return to the bonds. In equilibrium, the holdings of bonds is zero. e^{b_t} represents a discrepancy between the interest payment received by the household on financial asset and that determined by the central bank. It plays a similar role as the equity shock which influences financial asset premium described in Christiano, Motto and Rostagno (2013). A positive change of this shock increases the return to capital and thus investment. The shock follows:

$$b_t = \rho_b b_{t-1} + \varepsilon_t^b, \varepsilon_t^b \sim N(0, \sigma_b^2)$$

There is a cost associated with capital utilization for each unit of capital rented, denoted as $P_t a(u_t)$ in units of final goods. It is assumed that $a(1) = 0$, a' and $a'' > 0$. The higher the utilization rate, the more the output and thus the consumption; on the other hand, the higher the cost of the use of capital. Π_t is any profit earned by intermediate goods firms and labor union that is transferred to households.

The evolution of the effective capital is

$$k_{t+1}(j) = (1-\delta)k_t(j) + e^{q_t}\left[1 - S\left(\frac{Y_t^I(j)}{Y_{t-1}^I(j)}\right)\right] Y_t^I(j)$$

δ is capital depreciation rate. $S(\cdot)$ is investment adjustment cost, with the functional form $S\left(\frac{Y_t^I(j)}{Y_{t-1}^I(j)}\right) = \frac{\kappa}{2}\left(\frac{Y_t^I(j)}{Y_{t-1}^I(j)} - \Lambda\right)^2$. In steady state, $S(\Lambda) = S'(\Lambda) = 0, S'' > 0$. As mentioned above, q_t is the IST shock. Since q_t is unobservable, the effective capital k_t is also unobserved in this economy.

In equilibrium, the choices of labor, capital utilization, consumption, investment and bond holding will be the same across all households.

2.5 Labor union

Households supply labor to a competitive labor packer before production occurs. The labor packer aggregates the differentiated labor from households according to

$$n_t = \left[\int_0^1 n_t(j)^{1-\frac{1}{\nu_t^w}} dj\right]^{\frac{\nu_t^w}{\nu_t^w - 1}}$$

where n_t is the aggregate labor demand by intermediate goods firms. ν_t^w is the elasticity of substitution between differentiated labor, capturing the wage markup shock, which affects the demand of differentiated labor. Define $\frac{\nu_t^w}{\nu_t^w - 1} = 1 + \vartheta^w e^{\vartheta_t^w}$ where ϑ_t^w follows:

$$\vartheta_t^w = \rho_w \vartheta_t^w - \varpi_w \varepsilon_{t-1}^w + \varepsilon_t^w, \varepsilon_t^w \sim N(0, \sigma_w^2)$$

The labor packer maximizes profits subject to the above demand function. From its first order condition, the labor demand function for each household is:

$$n_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\nu_t^w} n_t$$

where n_t is the aggregate labor demand. The aggregate wage is obtained by combining the labor demand with zero profit condition as

$$W_t = \left[\int_0^1 W_t(j)^{1-\nu_t^w} dj\right]^{\frac{\nu_t^w}{1-\nu_t^w}}$$

Labor union is the channel through which labor packer combines labor from households. Labor union has monopoly power and sets the optimal wage following Calvo (1983) rule. In particular, at each period, a fraction $1 - \theta_w$ of households' wages are changed. All the other households can only partially index wages by inflation. Therefore, the labor union chooses $W_t^*(j)$ to maximize:

$$\max_{W_t^*(j)} E_t \sum_{s=0}^{\infty} (\theta_w \beta)^s \frac{\lambda_{t+s}}{\lambda_t} \left\{W_t^*(j)(\Pi_{\tau=1}^s \pi_{t+\tau-1}^{\iota_w} \pi_*^{1-\iota_w}) n_{t+s}(j) - \psi W_t n_{t+s}(j)\right\}$$

subject to

$$n_t(j) = \left[\Pi_{\tau=1}^s \pi_{t+\tau-1}^{\iota_w} \pi_*^{1-\iota_w} \frac{W_t^*(j)}{W_t}\right]^{-\nu_t^w} n_t$$

where ι_w is the parameter controlling indexation of wage on inflation.

Differentiate the equation with respect to the wage, $W_t^*(j)$, and the optimal wage is obtained as:

$$E_t \sum_{s=0}^{\infty} (\theta_w \beta)^s \frac{\lambda_{t+s}}{\lambda_t} n_{t+s} \left[W_t^*(j)(\Pi_{\tau=1}^s \pi_{t+\tau-1}^{\iota_w} \pi_*^{1-\iota_w}) - \frac{\nu_t^w}{\nu_t^w - 1} \psi W_t\right] = 0$$

Similar with the aggregate price, the aggregate wage evolves as:

$$W_t^{1-\nu_t^w} = \theta_w (W_{t-1} \pi_{t-1}^{\iota_w} \pi_*^{1-\iota_w})^{1-v_t^w} + (1-\theta_w)(W_t^*(j))^{1-\nu_t^w}$$

Note again that the aggregate wage and optimal wage are determined at the pre-agreed level based on the contract between labor union and intermediate goods firms. They may not coincide with the realized marginal products of labor or those under sticky wages and perfect information.

2.6 Monetary authority

To investigate the effects of stabilization policy under information friction, I assume that the central bank follows a certain Taylor rule such that the dynamics of money supply is to achieve the target level of nominal interest rate, taking the form of:

$$\frac{R_t}{R^*} = \frac{R_{t-1}^{\varphi_R}}{R^*}\left[\left(\frac{\pi_t}{\pi^*}\right)^{\varphi_\pi}\left(\frac{Y_t}{Y_t^*}\right)^{\varphi_y}\right]^{1-\varphi_R}\left[\frac{Y_t/Y_{t-1}}{Y_t^*/Y_{t-1}^*}\right]^{\varphi_{dy}} e^{m_t}$$

where R^* and π^* are the steady state values of the interest rate and inflation rate respectively. Y_t^* is the level of real output at period t, defined as the output level under the economy with flexible prices and wages and uncertainty, and without markup shocks. φ_π and φ_y are elasticities of the interest rate with respect to inflation and output gap respectively. φ_{dy} is the elasticity of interest rate with respect to the change of output gap as in Woodford (2003). φ_R introduces interest rate smoothing to match the path observed from US data. e^{m_t} represents monetary policy shock. which follows

$$m_t = \rho_m m_{t-1} + \varepsilon_t^m, \varepsilon_t^m \sim N(0, \sigma_m^2)$$

One interesting question is whether the monetary policy has different impacts on the economy under information friction compared to perfect information, and whether this comparison is sensitive to different expectations about the underlying fundamentals. Intuitively, the existence of uncertainty could deviate agents' expectations from the true values of the unobserved fundamentals, which would alter the dynamics of output and inflation, because agents' behavior are no longer directly dependent on the true values of the fundamentals as assumed in standard DSGE framework, but on the expectations about the current state of the economy. Therefore, a policy may have different impacts on the economy if it follows the same rule chosen under certainty. It is important for the monetary authority to consider different possibilities when it faces uncertainty. Moreover, the design of optimal monetary policy may also be influenced, since the efficient economic state to achieve and the distortions to offset may differ due to information friction. Monetary authority will have to adjust policies accordingly so that it can achieve a desired welfare and aggregate volatility. I will address these questions in more details in section 4.

Under this Taylor rule, the interest rate responds to deviation of inflation from its steady state, as well as to the that of output gap, and a monetary policy shock. The implementation of the target interest rate is through open market operations which are financed by the lump-sum tax T. The tax also finances the government spending G. The balanced budget constraint requires:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$

Define e^{g_t} as the ratio of government spending to the steady state output, where g_t follows:

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g, \varepsilon_t^g \sim N(0, \sigma_g^2)$$

2.7 Bayesian learning process

Economic agents, including households, labor union, firms and monetary authority, have the information set and learning behavior described as follows:

Before observing signals

At the beginning of each period t, the prior beliefs on the distribution of effective capital, TFP and IST, denoted respectively as g_t^K, g_t^Z, g_t^Q, are given[4]. They are formed by the end of last period. The investment decision from period $t-1$ affects the actual capital stock and output in period t. It also affects the expected effective capital in period t in a way disciplined by the evolution of beliefs. The labor decision, due to information friction, is naturally different from that under standard New Keynesian models. In the latter, the TFP shock is realized and observed at the beginning of each period; households choose labor according to the certain substitution rate between marginal benefit of consumption and leisure. Under uncertainty, since households cannot observe the actual TFP shock and effective capital, it is impossible to make the decision based on the exact substitution. Instead, households choose labor based on the wage contract with labor union as described before. The choice of labor before observing signals is assumed to affect both the utility and the output. The utilization rate of the effective capital is also chosen before any signals happen based on the contract on rental rate between firms and households.

Regarding the underlying processes of the fundamentals, at the beginning of each period t, all shocks, including the TFP and IST shocks are realized, but some are unobserved. There are two signals uncovered after production. One signal is the realized output Y_t, a function of the realized but unobserved TFP shock z_t, the unobserved predetermined effective capital k_t, the capital utilization rate u_t and the labor input n_t, following

$$Y_t^s = \left[\int_0^1 Y_t(i)^{1-\frac{1}{\nu_t^p}} di\right]^{\frac{\nu_t^p}{\nu_t^p-1}} = \left[\int_0^1 \left(e^{z_t}(u_t k_t(i))^{\alpha} n_t(i)^{1-\alpha}\right)^{1-\frac{1}{\nu_t^p}} di\right]^{\frac{\nu_t^p}{\nu_t^p-1}}$$

The other signal ϕ_t gives information on the realized IST shock, as a function of its own lagged value ϕ_{t-1}, IST shock q_t and an error term v_t.

$$\phi_t = \rho_\phi \phi_{t-1} + (1-\rho_\phi)e^{q_t} + e^{v_t}, v_t\tilde{\ } N(0, \sigma_v^2)$$

The lagged value ϕ_{t-1} is introduced to capture the idea that the signal could be persistent, measured by ρ_ϕ between 0 and 1. When ρ_ϕ is close to 0, the signal does not persist and previous signals have less weight for the information. When ρ_ϕ is close to 1, the signal contains little information regarding current IST shock q_t. v_t is a disturbance in the signal process, following an i.i.d normal distribution. As mentioned in the Introduction, this disturbance has a similar

[4] For learning process, $m()$ denotes posterior beliefs, and $g()$ denotes prior beliefs. Capital notations such as Q, Z, K Y means random variables, whereas small notations such as q, z, k, y means the realized values of the corresponding random variables.

role to the "sentiment" shock in the literature, which can capture agents' attitude towards the underlying fundamentals, including the investment technology. When there is a positive shock v_t, the signal will increase and the expected IST may also increase because agents may believe that the observed increase in ϕ_t is at least to some extent attributed to the increase of q_t, although what actually happens is solely an increase in v_t. As a result, over optimistic agents increase investment and thus output. However, when the true economic state is gradually learned, output will decrease and may even fall below the under-steady-state level, resulting in recession. This fluctuation does not involve any initial changes of fundamentals.

After observing signals

After observing the two signals, economic agents are able to update their common knowledge of the effective capital stock, TFP and IST following Bayes' rule, given prior beliefs. In addition, knowing the evolution of the aggregate productivities following AR(1) processes, agents can derive prior beliefs about their values for period $t+1$. Knowing the evolution of the effective capital stock, agents can derive prior beliefs about the effective capital for period $t+1$, given the choice of Y_t^I. The expectation of Y_{t+1},(the likelihood of Y which is conditional on prior distributions of the unobserved variables for period $t+1$) is formed according to the known production function. Similar rules apply to ϕ_{t+1}. The detailed calculation of beliefs' evolution are demonstrated in Appendix 2.

2.8 Aggregation

First, the aggregate demand in the economy is obtained by combining the households' and government budget constraints. The aggregate demand in the economy is

$$Y_t^d = c_t + Y_t^I + G_t + a(u_t)\hat{k}_t$$

The aggregate supply is

$$Y_t^s = \left[\int_0^1 Y_t(i)^{1-\frac{1}{\nu_t^p}} di \right]^{\frac{\nu_t^p}{\nu_t^p - 1}} = \left[\int_0^1 \left(e^{z_t} (u_t k_t(i))^{\alpha} n_t(i)^{1-\alpha} \right)^{1-\frac{1}{\nu_t^p}} di \right]^{\frac{\nu_t^p}{\nu_t^p - 1}}$$

It is not possible to simplify this expression since input usages across firms differ. However, the linear aggregation $\int_0^1 Y_t(i)di$ is approximately equal to Y_t within a local domain of the steady state. Hence for local analysis applied to the estimation (including learning process) and simulation, we can simply use[5]

[5] We will also use this approximation for the learning process.

$$Y_t = \int_0^1 Y_t(i)di = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\nu_t^p} Y_t di = \frac{e^{z_t}(u_t k_t)^\alpha n_t^{1-\alpha} - \Phi}{\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\nu_t^p} di}$$

In equilibrium, $Y_t^d = Y_t^s$

Similarly, for the labor market, labor demand equals to labor supply in equilibrium.

$$n_t^d = \frac{n_t}{\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\nu_t^w} dj}$$

3 Numerical analysis

3.1 Solution strategy

The dynamics of the economy depends on the evolution of the underlying fundamentals, some of which are unobserved and therefore their distributions matter. In this framework, the subjective beliefs regarding IST, TFP and effective capital serve as state variables. As pointed out by Lorenzoni (2009), when state variables involve high order dimension, such as distribution, certainty equivalence does not hold and the paths of optimal choices depend not only on the first moments, but also on the higher moments of the density. In order to overcome the curse of dimensionality and the complication of functional derivatives which give no explicitly tractable solution, I use a methodology similar to that developed in Ma and Samaniego (2015), which is based on the first order approximation around certain steady state. This method enables to characterize the dynamics of the economy by using only the evolution of the linearized variables instead of the original variables. Through the log-linearization, it is possible to transform the Bayesian learning process to be linear and Gaussian under some loose initial conditions, and to implement the calculation of the linear Gaussian Markov process through the Kalman filter. In this way, only mean and variance matter for the dynamics. The detailed calculation is shown in Appendix 3. Here are the key equations characterizing the evolution of the linearized unobserved variables:

$$\begin{aligned}
E_t z_t &= E_{t-1}z_t + P_{1,1}(\frac{\tilde{Y}_t}{\Phi_p} - (1-\alpha)\tilde{n}_t - \alpha u_t - E_{t-1}z_t - \alpha E_{t-1}\tilde{k}_t) + P_{1,2}(\tilde{\phi}_t - \rho_\phi \tilde{\phi}_{t-1} - E_{t-1}q_t) \\
E_t \tilde{k}_t &= E_{t-1}\tilde{k}_t + P_{2,1}(\frac{\tilde{Y}_t}{\Phi_p} - (1-\alpha)\tilde{n}_t - \alpha u_t - E_{t-1}z_t - \alpha E_{t-1}\tilde{k}_t) + P_{2,2}(\tilde{\phi}_t - \rho_\phi \tilde{\phi}_{t-1} - E_{t-1}q_t) \\
E_t q_t &= E_{t-1}q_t + P_{3,1}(\frac{\tilde{Y}_t}{\Phi_p} - (1-\alpha)\tilde{n}_t - \alpha u_t - E_{t-1}q_t - \alpha E_{t-1}\tilde{k}_t) + P_{3,2}(\tilde{\phi}_t - \rho_\phi \tilde{\phi}_{t-1} - E_{t-1}q_t)
\end{aligned}$$

where $E_t z_t, E_t q_t$, and $E_t \tilde{k}_t$[6] are the posterior beliefs about TFP, IST and (log linearized) effective capital respectively at period t. $E_{t-1} z_t, E_{t-1} q_t$, and $E_{t-1} \tilde{k}_t$ are their corresponding prior beliefs formed by the end of period $t-1$. $\tilde{Y}_t$ is the log linearized aggregate output. Φ_p is the gross share of fixed cost in the steady state production, defined as $\Phi_p = \frac{Y*+\Phi}{Y*}$. $\tilde{n}_t$ is the log linearized labor input. The matrix P is calculated as a stable solution to the Algebraic Riccati equation, also referred to as the stable optimal Kalman gain. It is a function of the (stable) covariance matrix for z, q and $\tilde{k}$ and the parameters capturing the persistence of z, q and ϕ, as well as the production parameter α. P measures the dependence of posterior beliefs on different sources of information. A larger P indicates the updating of beliefs relies more on the corresponding information. Without uncertainty, P will be a zero matrix and the prior and posterior beliefs will be the same. Agents are affected by the accuracy of their initial expectations and the learning process (filtering information captured by P) on the current state of the economy. Note that the disturbances to z, q and ϕ can influence the expectation of all three unobservables. Moreover, incorrect beliefs about one unobserved variable can be transmitted to others under uncertainty, even if there are no shocks hitting the economy. These changes in beliefs could create fluctuations that would not happen under certainty.

3.2 Estimation

I use Bayesian estimation techniques to estimate the model outlined in the previous section. The Bayesian methodology has become a popular method to characterize the posterior properties of structural parameters in medium scale DSGE models. In the following, I will discuss the data used to estimate the model, the specification of the priors, estimation results, and model's fit.

3.2.1 Data

The model is fitted to quarterly postwar U.S. data on output, consumption, investment, hours worked, inflation and nominal interest rate from 1947QIII to 2006QIV. I also use an alternative dataset which includes expected output in addition to the above mentioned variables, which starts from 1969QIII, the first quarter in which the Survey of Professional Forecasters has data of expected real GDP. 2006QIV is the last quarter before the starting of financial crisis, after which the policy rate hits the zero lower bound soon. All the original data of the economic variables are extracted from the National Income and Product Accounts (NIPA) dataset. The expected output data is extracted from the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia. Output corresponds to the real GDP per capita, which is obtained by dividing the nominal GDP per capita by the chain-weighted deflator for GDP. Consumption is obtained as the consumption expenditures on nondurable goods and services. Investment is obtained by the sum of gross domestic private investment and consumption

[6] Variables with tilt are defined as $\tilde{X}_t = \log X_t - \log \bar{X}$ where $\bar{X}$ is the steady state of X

expenditure of durable goods. Hours worked is obtained as the log of hours worked in the non-farm business sectors. I obtain inflation by taking the annualized percentage change in the GDP deflator. Nominal interest rate corresponds to the Federal Funds Rate in percent. Finally, the expected output corresponds to the median forecasters' one quarter ahead expected real GDP reported in the Survey of Professional Forecasters divided by population.

In order to solve and estimate the model, the system of equations characterizing the equilibrium are transformed to the log deviation from their certainty steady states. Therefore, it is necessary to transform the observed data (except for inflation rate and nominal interest rate) to the percentage deviation from the stochastic trend, obtained by the HP filter with parameter 1600 for quarterly frequency. The generated data have the same conceptual meaning with the variables in the model. For inflation rate, the trend is obtained by the mean of inflation over the data periods. For nominal interest rate, since it is reported as the net rate in percentage points and annualized form but the model is written in quarterly frequency and has gross rates, it has to be transformed into a quarterly gross interest rate first by dividing the data by four hundred and adding 1, yielding the observed time series of nominal interest rate in the model. The correspondence between the data on the left hand side and the measurement equation from the model on the right hand side is:

$$\begin{bmatrix} \log GDP_t - \log GDP \\ \log CON_t - \log CON \\ \log INV_t - \log INV \\ \log HOUR_t - \log HOUR \\ \log INT_t - \log INT \\ FFR_t \\ (\log E_t GDP_{t+1} - \log E_t GDP) \end{bmatrix} = \begin{bmatrix} \tilde{Y}_t \\ \tilde{c}_t \\ \tilde{Y}_t^I \\ \tilde{n}_t \\ \tilde{\pi}_t \\ 100(\log R_* + \tilde{R}_t) \\ (E_t \tilde{Y}_{t+1}) \end{bmatrix}$$

where GDP_t, CON_t, INV_t, $HOUR_t, E_tGDP_{t+1}$ are respectively the data for real GDP per capita, consumption, investment, hours worked and expected real GDP as described above, and GDP, CON, INV, $HOUR, E_tGDP$ their corresponding trends generated by HP filter. $\log INT_t - \log INT$ is the deviation of interest rate from its trend defined as $\log(mean(INF_t^{data}))$. FFR_t are transformed nominal interest rate defined as $FFR_t = \log(1 + \frac{FFR_t^{data}}{400})$. R_* is the steady state nominal interest rate.

3.2.2 Priors

I choose a set of priors that are relatively wide and broadly in line with those used in the literature (e.g. Smets and Wouters (2007) and Justiniano et al (2011)). The priors for the persistence of stochastic shocks and markup process are rather non informative, assumed to follow beta distribution with mean 0.5 and standard deviation 0.1. The priors for the standard deviation of disturbances to the shocks follow inverse-gamma distribution with mean 0.1 and standard deviation 1. The Kalman gain parameters are deep parameters in the sense that they depend on the choices of other parameters, so there is no priors set for them but just the posteriors are reported. The priors for the other parameters are the same as in Smets and Wouters (2007) benchmark model.

I fix a small number of parameters which are known in the literature to be difficult to identify in DSGE models. The depreciation rate δ is fixed at 0.025 and the exogenous steady state government spending g at 22% which corresponds to the government spending-GDP ratio for the sample period. The labor elasticity (inverse Frisch elasticity) is set at 3. Microeconometric studies tend to estimate a lower value (0 to 0.5) than that by macroeconomists (2-4) in a general equilibrium framework. The choice in this paper is based on Peterman (2014) which tries to reconcile the Micro and Macro estimates of this elasticity.

Table 1 and 2 summarize the priors and the estimates of the structural parameters and shock processes respectively.

3.2.3 Estimation results

Given prior distribution and the observation variables, I estimate the posterior distribution and the mode of the structural and shock parameters in the model by maximizing the log posterior likelihood and then using the Metropolis-Hastings Monte Carlo technique to obtain a complete posterior distribution of the parameters. I estimate both models with and without information friction using the same dataset. And I use the dataset with and without forecasted variables for the estimation but find it does not generate much difference. Therefore, I only show the results from the estimation based on the data including the expectation variable which uses a shorter sample period between 1969 and 2006. The estimates of the posterior mode and standard deviation of the structural parameters and shocks are shown in Table 1 and 2. The log likelihood of the certainty benchmark estimation is -1689.92, the absolute value of which is higher than that under uncertainty using the same dataset. This implies to some extent that the model with uncertainty is able to fit data better than the standard DSGE framework[7].

Some of the parameters are worth discussing. The capital share parameter, α is only 0.198 and smaller than the common assumption of 0.33. The degree of wage and price stickiness are higher than 0.5, at 0.766 and 0.518 respectively. This implies the average duration of wage contract is 4 quarters and that of price is 2 quarters. The latter is a bit lower than other estimations in the literature where the price duration is 3 quarters on average. The Taylor rule smoothing parameter φ_R of 0.88 is high, indicating that during the sample periods, the monetary policy rule followed the lagged interest rate closely, and on average did not change substantially. The Taylor rule parameters on output gap and its change, φ_y and φ_{dy} are around 0.2, smaller than that on inflation of 1.87. This indicates that the long run reaction to inflation in the Taylor rule is much stronger than that to the output. In the Taylor rule, inflation is an important factor that shapes the central bank's policy.

[7] Most of the parameters estimated are well identified as shown in Table 3.

Regarding the stochastic shocks that characterize the dynamics of the economy, the persistence of TFP, government spending, wage markup and price markup are all very high. This implies that most of the forecast error of variances in the long run will be explained by these four shocks. On the other hand, both the financial asset premium shock and monetary policy shock have low persistence. The IST shock has a larger than 0.5 persistence, which is in line with the business cycle literature. The unique parameter ρ_v, which captures the persistence of the signal regarding the IST and market sentiment, is 0.342. This means that the persistence of the signal driven by the IST shock is weaker than by the sentiment shock. The variance of government spending shock and IST shock are among the highest in the estimates. This is consistent with the observance that government spending and investment are more volatile than other macroeconomic variables in the sample periods. Overall, the data is informative in the stochastic processes of the exogenous shocks and the dynamics of the framework, as the posterior densities of the structural and shock parameters are considerably different from their priors.

3.3 Model's fit

In last section, it is briefly mentioned that this framework with uncertainty performs better than the certainty counterpart by comparing the marginal likelihood obtained from the estimation of the two models. In this section I compare the model's generated statistics with those from the actual data and certainty framework, as well as the pseudo out-of-sample forecast performance with that based on vector autoregression (VAR) estimation.

3.3.1 Within-sample performance

I calculate the standard deviation and correlation with output (in absolute value) of the targets used for estimation based on the data and the estimation results from uncertainty and certainty framework. The comparison are shown in Table 4. For the statistics estimated from the models, I show the median values. In particular, compared with data, the model with uncertainty overpredicts the standard deviation of consumption and output, whereas underpredicts those of investment, labor, interest rate and inflation. Compared with the predictions from the certainty model, the model under uncertainty in general matches better with data, even though the correlation between consumption and output, and investment and output are better predicted by the certainty model, but not overperfoming the uncertainty model to a substantial extent. This result is overall consistent with the marginal likelihood comparison that the certainty model characterizes the economy better than the standard DSGE model.

One thing to note is neither model could perfectly match with data even though various stochastic shocks have been included in the models. An important reason for this is that the Bayesian estimation technique tries to match the model prediction with the covariance of the data and the first order and higher order moments. It has to balance the fit across these statistics. Even with various stochastic shocks and uncertainty in the model specification, it is still unlikely to match well all the statistics for selected variables. Nonetheless, the comparison provides an insight on the overall accepted within-sample performance of the model.

3.3.2 Forecast performance

The previous section compares the within-sample performance of the model. In this section, I further calculate the out of sample prediction performance of the model and compare it to that from the VAR and certainty framework respectively. The forecast performance is measured with marginal likelihood. In addition, the pseudo out-of-sample predicted mean squared errors for the observation variables are also calculated for robustness check.

Table 5 demonstrates the marginal likelihood values of the uncertainty, certainty, and unconstrained VAR models with different numbers of lags. These models are all estimated over the entire sample period. From the comparison, it is not surprising that both DSGE models generate better predictions than the unconstrained VAR with the same observation variables but different lags[8]. More importantly, as mentioned before, the DSGE model with uncertainty overperforms the certainty benchmark.

Table 6 shows the predicted mean squared errors for selected variables in different horizons over the period 1990QI:2006QIV. The VAR(1) and DSGE models shown in the table are estimated using data 1969QIII:1989QIV. The percentage gain (+) or losses (-) are calculated as the difference between the log of the forecasted error covariance from the VAR(1) model and the DSGE models. The results reassure that the DSGE models tend to perform better than VAR model.

3.4 Business cycle properties

Standard RBC or New Keynesian models are successful to some extent to explore the factors that contribute to US business cycles in the post World War II US economy. However, as recent literature shows, the feature of uncertain economic state also plays an important role in characterizing the economy and should be included in the specification of the general equilibrium models. In this paper, I introduce the information friction regarding the current state of the economy to an otherwise standard New Keynesian model, which enables discussion of the role of uncertainty and its implications on business cycles. For example, what are the main driving forces of macroeconomic fluctuations? Is IST an important factor accounting for the dynamics of investment and output? What is the role of "sentiment"? What determines inflation and output gap historically? With the model proposed in this paper that has been shown to perform better than the standard DSGE model in characterizing the economy, I am able to contribute to the debate of these central questions in the literature.

In order to investigate the roles different shocks play in characterizing the US business cycles, I use the traditional forecast variance decomposition for the model-generated fluctuations

[8] It is known that the unconstrained VAR as a representative of overparameterized models is typically not able to generate sound predictions. See, for example, Smets and Wouters (2007).

at business cycle frequencies. The decomposition shows the contribution of different stochastic shocks to the variances of macroeconomic variables. The result is shown in Table 7.The first thing to note is that the TFP shock is an important factor in explaining the fluctuations of output, consumption, investment and labor, although it may not be the most important factor explaining the variances of those variables. The second to note is the role of the IST shock. It explains a fair amount of fluctuations in investment and output. Quantitatively, this result lies between the findings from Justiniano et al. (2011) in which the IST shock accounts for almost zero percent of output, consumption, investment and hours worked, and the findings of Greenwood et al. (2000) and Fisher (2006) that the IST shock is an important contributor to business cycles. One of the reasons mentioned in Justiniano et al. (2011) to explain the different conclusions on the importance of the IST shock between their paper and Fisher (2006) is whether to use the data on the relative price of investment directly to match IST in the model. This paper tries both methods and does not find much difference on the estimation. Therefore, this evidence proposes another story about why IST might be important (or not): it depends on the accessibility of information about the technology. If economic agents are unable to observe IST, as assumed in this paper, and only gradually recognize its values, they behave conservatively when a positive IST shock hits. As a result, the fluctuations of investment due to the change of IST are not as big as without information friction. As for the equity premium shock, it explains a fair portion of volatility in output, consumption, investment and hours worked. This evidence shows that the financial shock is an important demand side factor, consistent with findings from other papers focusing on its role in recent financial crisis[9]. The last is the sentiment shock, which captures the undue optimism or pessimism about the investment technology. On average, it is a non negligible factor that contributes to business cycle fluctuations. This result is between the findings from Barsky and Sims (2011) which claims that solely having sentiment shock is difficult to generate quantitatively large economic fluctuations, and Angeletos and La'O (2013, 2015) which demonstrate that the "sentiment" is an important driving force of economic activities. The last two variables in the table are the beliefs about IST and TFP respectively. Both sentiment shock and IST shock contribute to the volatility of the expected IST. It means when an IST shock or sentiment shock hits, it takes time for economic agents to differentiate between the two shocks and therefore, even though the sentiment shock itself is non persistent, the influence on the macroeconomic variables could be persistent. This is why investment is also influenced by the sentiment shock, for the decision of investment is based on the beliefs about IST, which in turn is affected by the sentiment shock. By contrast, the volatility of the expected TFP is largely dependent on the TFP shock, indicating that agents could learn relatively quickly when a TFP shock hits, even though both TFP and IST shocks are unobserved.

As for historical evidence, Figure 1 shows the historical variance decomposition for inflation between the initial period of the sample to 2006QIV. Throughout this period, wage markup

[9] For example, see Christiano, Motto and Rostagno (2013).

shocks are the most important driver of the fluctuations of inflation. Monetary policy shocks, on average, play a small role. However, in the late 1970s and early 1980s, they did contribute to the disinflation during the Volcker period. The two unobserved shocks, IST and TFP shocks, also play a non negligible role, especially in the 1970s and 2000s. Figure 2 shows the decomposition for output gap during the same period. Wage markup, equity premium, IST and monetary policy shocks all contribute to the dynamics of output gap significantly. Notably, TFP shocks do not seem to play a role. Finally, it is clear that in the late 1970s, monetary policy shocks account for the recovery of the economy to a large extent.

These findings suggest that the inclusion of uncertainty could alter the explanation of business cycles compared to that under certainty. More importantly, uncertainty could serve as a channel through which an otherwise ignorable disturbance, such as the change in the "sentiment", affects the economy. It is therefore important for the monetary authority to acknowledge different scenarios of economic environment and their implications on monetary policies, which will be elaborated in next section.

4 Monetary policy implications

The information friction that temporarily decouples agents' decisions from economic fundamentals but connects them to their expectations creates a different environment for policy-makers. In this section, I will address three key questions to understand monetary policy under uncertain economic state: i) Does a routine stabilization policy rule commonly used affect the dynamics of macroeconomic variables differently? ii) How could the central bank make optimal policy under information friction? and finally, iii) What are the welfare effects of different policy actions?

4.1 Taylor rule effects

In order to illustrate the monetary policy's stabilization effects under uncertainty, i.e., how policy accommodates or offsets various shocks when some supply and demand shocks are not observed, I explore the impulse responses of macroeconomic variables to stochastic shocks under the Taylor rule introduced before. The responses are expressed as percentage deviation from the steady state. The parameters used in the exercise are estimated based on the model with uncertainty. In the simulation, one period starts with the observation of shocks and/or signals, and ends by the observation of next shocks and/or signals. Initially, the economy is assumed to be in a "correct" steady state where all variables are in the steady state values of zero and all the expectations are accurate and equal to the true fundamentals. Then, shocks are realized and signals are observed. The figures show the behavior of macroeconomic variables after observing the shocks and/or signals. The responses of beliefs on productivities are the prior beliefs for the next period.

4.1.1 Observed government spending shock

Figure 3 depicts the effects of one standard deviation shock on government spending. Both models, with and without information friction, generate similar paths of persistent responses of macroeconomic variables. When there is a positive shock on government spending, there are immediate increases in output, labor supply, and inflation, but consumption and investment decrease due to the crowding out effect. The reason why uncertainty does not influence the pattern of the behavior is: i). It is assumed that the economy starts at a correctly perceived steady state. At the initial period, agents' beliefs coincide with the true values of economic fundamentals, and ii). Economic agents can observe the occurrencce of government spending shock and their beliefs about the unobserved fundamentals are not affected by this shock. Consequently, the responses are similar even though they cannot observe some underlying fundamentals. Similar results apply to other scenarios when the shocks hitting the economy are observable.

As shown in Figure 3, the responses of monetary policy are similar under both environments. The higher output and inflation oblige the central bank to increase nominal interest rate to offset the positive demand shock of an increase in government spending. The implication for monetary policy is, if the disturbances that can cause fluctuations in the economy is easy to be identified and differentiated by agents, an otherwise effective monetary policy will play a similar role to stabilize the economy. There is no need for the monetary authority to resort to more complicated or sophisticated decision process.

4.1.2 Observed monetary policy shock

Figure 4 shows the responses of economy to a one standard deviation shock on monetary policy at the level of 0.23. The policy shock generates a very persistent response in output. The peak effect occurs roughly one year after the shock and the negative effects on output last for over three years. There are also persistent decreases in consumption, investment, and inflation. The model also illustrate the dynamic responses of interest rate to a monetary policy shock. The contractionary policy shock induces a sharp increase in interest rate which then returns to its pre-shock level within a year. The model does account for the overshooting pattern of the interest rate empirically found in the data. Note that the initial increase of the nominal rate is less than 0.23, due to the downward adjustment induced by the decline in inflation and output gap[10]. The response of real rate is larger than that of nominal rate as a result of the decrease in expected inflation. It is obvious from the comparison that the dynamics of the economy is almost the same under uncertainty and certainty. This is not surprising because the monetary policy shock can be observed by private sectors, just as the government spending shock, and an observed shock doesn't deviate agents' (correct) expectations about the unobserved fundamentals. Therefore, the existence of information friction does not magnify or

[10] The response of output corresponds to that of the output gap, because the output level under flexible prices and wages is not affected by the monetary policy shock.

mitigate monetary policy shocks. In general, the monetary authority is safe to expect policy effects in the direction of its desired influence on the economy.

4.1.3 Unobserved TFP shock

Figure 5 depicts the effects on economic variables after a positive shock on TFP. Different from the responses to the observed shocks, when an unobserved shock hits the economy, the responses of economic agents may differ. Since agents do not observe the occurrence of the TFP shock but only observe an increase of output, they attribute some changes of the higher output to higher effective capital, which could be influenced by previous IST. As a result, their beliefs on IST increase slightly initially. Output responds more to a positive TFP shock, as the choice of labor decreases less. Consumption and investment also respond slightly stronger than under certainty. Inflation is slightly higher at the beginning periods. The information restricted Phillips curve suggests that inflation is dependent on the expected current and future marginal costs. Under a positive TFP shock, since the expected output is less than the realized output, the expected marginal costs are thus smaller than the realized marginal costs. Therefore firms that have the opportunity to reset prices choose a price level relatively higher in order to realign their markup closer to the desired level.

Over all, however, the responses of macroeconomic variables are not significantly different. This is because of the quick learning about the TFP shock. The expected TFP is 18% lower than the actual one at the initial period, but the error reduces to 0.4% at the 4th quarter after the TFP shock hits. Agents learn most of the initial variation in TFP within a year. On the other hand, the error between the expected and actual IST (which is 0) is never bigger than 0.4%, with the highest of 0.3% at the initial period. This evidence implies that agents are able to learn quite quickly and accurately about the changes in the TFP shock. This is because the signal from which agents extract most information about the TFP shock, the output, increases immediately when the TFP shock hits, whereas the signal ϕ that provides more information for IST does not change. Therefore, even though the TFP shock is unobserved, its effects are similar compared to the certainty environment.

In both cases, the increase of TFP is accommodated by the central bank, which lowers nominal and real interest rates. This shows that, even if not being able to observe the occurrence of the supply side TFP shock, the central bank can still use an otherwise effective Taylor rule to achieve its desired impact on the economy, as a result of agents' quick learning of the unobserved fundamentals. Moreover, the Taylor rule is slightly more effective in accommodating a positive supply shock because the decline in inflation is persistently smaller in the short run under uncertainty.

4.1.4 Unobserved IST shock

Figure 6 depicts the effects on economic variables after one standard deviation shock on IST. Economic agents behave more conservatively compared to the certainty framework, and the difference in the responses are more significant than the previous scenario. Even though output and inflation increase in response to a positive IST shock, the magnitude is much smaller, because the expected increase in IST is smaller than the actual change of 0.6. Moreover, the expected TFP declines even though there is no TFP shock. As a result, the initial increase in output is only half of that under certainty. Moreover, the significantly weaker responses are also persistent due to that agents learn slowly when the IST shock hits. On impact, the expected IST is 77% lower than the actual one, and the error only declines to 12% at the 4th quarter and 2% at the 8th quarter. It takes 3 years for the expectation error to reach 0.4%, which is accomplished much sooner when the TFP shock hits. The misperception on TFP is also bigger compared to that on IST when the TFP shock hits. It is still 1.5% at the 12th quarter. The lower and slowly adjusted expectation on both IST and TFP contribute to the weaker dynamics when the IST shock hits.

As for the Taylor rule effects, the increase of nominal interest rate shows that the central bank correctly offsets the demand side IST shock, which contributes to the decrease of inflation. This reassures what we find when the TFP shock hits, that is, even though the central bank cannot distinguish supply and demand shocks, the Taylor rule is still appropriate and effective to stabilize the economy. Under uncertainty, however, the policy is relatively less sufficient in that the increase in nominal interest rate is slow and only attains the peak at the 6th quarter. This is because the information friction generates a misperception on TFP too, which weakens the effects of offsetting. This evidence shows that the existence of uncertainty could generate different implications for policy effects, when economic agents don't learn quickly enough about the uncertain economic state.

4.1.5 Unobserved sentiment shock

Figure 7 shows the impulse responses of the economy facing a one standard deviation sentiment shock. Following an optimism shock, investment and output increase. Agents know that the observed signal is a combination of the IST shock and disturbance, but contribute the increase of ϕ to at least some increase in IST, even though the actual series remain unchanged. Due to the increase in the expected value of IST, agents consequently invest more. Since the process of ϕ is persistent, agents observe subsequent values of ϕ above its steady state even though the sentiment shock last for only one period. As a result, they persistently have wrong expectations on IST and TFP, which generate persistent responses. As agents obtain more information, they realize the initial shock is less likely to be the IST progress. This influences their investment and utilization of capital. The latter decreases below the steady state after around one year. This is one mechanism that could explain what happens during a tech boom and bust cycle. An increase in market sentiment, for example, an optimistic expectation about the influence of a new technology may increase investment and capital utilization. Later, when agents gradually learn that it was only a tech bubble and no fundamental productivity has advanced, they adjust their beliefs on the investment technology and effectiveness of capital, which might cause the bust. If the sentiment varies dramatically during some particular periods, it could potentially generate considerable fluctuations in the short run[11].

The responses of nominal interest rate show that the Taylor rule plays a successful role to stabilize the economy. It increases to the peak within a year. As a result, inflation decreases and output gap closes quickly. The feature of the sentiment shock, however, poses a difficult question to the central bank. If the occurrence of the sentiment shock could be identified, policy actions should be offsetting in the short run since the unwarranted sentiment will return to steady state more quickly compared to the IST change. One tricky task is how to differentiate the IST shock from the sentiment shock, as the short run responses of the economy are similar. If contractionary monetary policy acts persistently, it risks to overshoot and contribute to the subsequent recession. Therefore, in practice, it is of particular importance for the central bank to obtain as much information before conducting policies when it suspects there is unwarranted change of market attitude towards the economic fundamentals.

4.1.6 Incorrect beliefs

The effects of the Taylor rule shown above are conditional on accurate beliefs about the economic state. It is natural to wonder what will happen during periods when economic environment changes dramatically and agents' beliefs are more likely to deviate from their true values. Will the same policy rule still be able to stabilize the economy? Figure 8 depicts the dynamics of the economy when the initial belief of IST deviates from its steady state at a level equivalent to the initial impact of a two standard deviation IST shock. An incorrect belief about IST leads to an incorrect belief about TFP. On thing to note is, even if no shocks happen to the economy, there are still responses in macroeconomic variables because the beliefs of IST and TFP deviate from steady state. This explains the increase of output gap and inflation. Regarding monetary policy, the nominal interest rate increases to offset the changes, resulting in an increase in the real interest rate. Inflation quickly decreases and becomes negative at the 6th quarter. The results suggest that the policy rule has persistent effects on both real and nominal variables. No matter whether the beliefs about the demand and/or supply shocks are correct, the Taylor rule is still effective to stabilize the economy.

The previous simulations have illustrated several important points. First, observed shocks have no significantly different effects on economic dynamics under uncertainty compared to certainty. Second, when unobserved shocks hit the economy, the responses of agents vary dependent on the differences in the convergence of beliefs to the underlying fundamentals.

[11] One thing to note is that, the historical decomposition, which is the best guess on what shocks contribute to the movement of macroeconomic variables in the history, shows that the sentiment shock doesn't contribute substantially to the dynamics of inflation and output gap. One potential reason is that the responses of economic variables are similar under the sentiment shock and IST shock, and it might be difficult to disentangle the contribution between the two in that exercise. With more restricted identification strategy, as shown in my other paper about the effects of unanticipated, news, and sentiment shocks on investment and output dynamics, I empirically identify the three shocks separately based on maximum forecast variance error method, and find that sentiment shock could influence output and investment to a substantial level in the short run.

The responses to an unobserved TFP shock don't vary significantly compared to certainty, whereas the unobserved shocks that are slowly learned by economic agents, such as the IST shock, will generate significantly different dynamics in the direction of less volatility. Third, incorrect beliefs alone can cause persistent responses with no shocks happening subsequently. Fourth and most importantly, the given Taylor rule is still effective in stabilizing the economy, even though the central bank cannot distinguish between the IST and TFP shocks, or more broadly between the supply and demand shocks. The last feature leads to interesting questions for the central bank to explore about the optimal policy under aggregate uncertainty, which will be the focus of the next section.

4.2 Optimal monetary policy

So far, I have focused on the implications of uncertainty on monetary policy effects illustrated by the Taylor rule. If the objective of the central bank is to stabilize the economy and maximize social welfare, how should the policy be designed when supply and demand shocks cannot be observed separately? What are the welfare implications for different policy actions? What if the central bank can influence the precision of information? I will discuss the characterization of the optimal monetary policy in this section.

4.2.1 Policy design

Compared to the efficient allocation determined by a benevolent social planner who seeks to maximize the representative household's welfare given its preference and realized stochastic shocks, in the proposed New Keynesian economy under information friction, the suboptimality of the allocation based on decentralized markets stems from three distortions. The first distortion is attributed to information friction, under which an inefficiently low level of output is generated. The second is the presence of market power due to the behavior of monopolistically competitive firms and labor unions, which enables the charge of higher prices and wages compared to perfectly competitive markets. The third distortion comes from the infrequent adjustment of prices by firms and wages by labor unions. As in literature, I assume that the central bank designs the optimal policy following what is proposed by the seminal work of Rotemberg and Woodford (1999), that is, minimizing the welfare loss function derived from a second-order approximation to the utility losses experienced by a representative household, by choosing federal funds rate and the coefficients capturing how this rate influences real variables.

First of all, under information friction, the central bank's welfare maximization problem is subject to the informational constraint that decisions are based on subjective beliefs rather than the actual fundamentals. This means that when making the optimal policy, the central bank can only use the information available but no superior knowledge on the unobserved underlying fundamentals. This distortion makes the optimal policy unable to generate the first best allocation, which could be obtained without information friction, but only able to achieve the allocation that is "constrained efficient."[12] The constrained efficient allocation is the one that would be obtained in an economy where the market power and the price and wage rigidities do not exist.

Second, since I am mainly interested in comparing the performances of different monetary policy actions, I assume that the central bank can eliminate the inefficiency from the market power by choosing an appropriate subsidy intervention to offset the first order distortion caused by the presence of monopolistic competition in the markets. This intervention can be easily relaxed and incorporated in the problem. I impose it in order to better understand the stabilization properties and welfare implications.

As shown in section 2, the firm's optimization problem in equilibrium is characterized by

$$\frac{W_t}{P_t} = \frac{E_t[Z_t Y_t]}{Y_t} \frac{(1-\alpha)Y_t(i)}{n_t}$$

Rearranging, we have

$$\frac{W_t}{P_t} = \frac{1}{E_t[M_p]} \frac{Y_t}{E_t[Y_t]} MPN_t$$

where M_p is the price markup, and MPN_t is the marginal product of labor. In addition, the substitution between labor and consumption derived from the utility maximization is:

$$-\frac{U_{n,t}}{U_{c,t}} \frac{E_t[M_w]U_{c,t}}{E_t[U_{c,t}]} = \frac{W_t}{P_t}$$

where M_w is the wage markup.

Without uncertainty, the optimal subsidy chosen by the central bank is to have $s = 1 - \frac{1}{M_p M_w}$ such that the optimal allocation of consumption and labor generates:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{M_w P_t} = \frac{(1-s)M_p W_t}{P_t} MPN_t = MPN_t$$

Under uncertainty, since the first best allocation cannot be achieved due to information friction, the choice of s should satisfy the condition to guarantee a constrained efficient state:

$$s_t = 1 - \frac{1}{E_t[M_p]E_t[M_w]}$$

Compared with certainty, it is obvious that the subsidy is dependent on the expectation of wage and price markups (marginal costs). In general, unless the expectations of the unobserved fundamentals coincide with their actual values, the optimal subsidy would be different from that under environment without information friction.

Given that the optimal subsidy completely offsets the market power distortion, the minimal welfare losses can be obtained by a policy that stabilizes the average price and wage rigidities.

[12] This notion is developed and analyzed in a broad class of quadratic games in Angeletos and Pavan (2007, 2009).

In particular, the central bank minimizes the average welfare losses experienced by households in the economy with sticky prices and wages in terms of consumption:

$$W = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma_c} [c_t - hc_{t-1}]^{1-\sigma_c} - \psi \frac{n_t^{1+\gamma}}{1+\gamma} \right\}$$

which yields

$$W = E_0 \sum_{t=0}^{\infty} \beta^t [\Gamma_y \tilde{Y}_t^2 + \Gamma_{\pi^p} (\pi_t^p)^2 + \Gamma_{\pi^w} (\pi_t^w)^2] + \Gamma$$

And the average period welfare losses can be expressed as a linear combination of the variances of the output gap, price inflation and wage inflation, given by

$$L = \Gamma_y var(\tilde{Y}_t) + \Gamma_{\pi^p} var(\pi_t^p) + \Gamma_{\pi^w} var(\pi_t^w)$$

where Γ_y, Γ_{π^p}, Γ_{π^w} are respectively the weights on output gap, price inflation and wage inflation, which are determined by the parameter values, and Γ are other terms that are independent of policy actions. Intuitively, under uncertainty, since the average (expected) volatility of output gap and inflation are lower than that under certainty when the TFP and IST shocks happen, and higher when the sentiment shock happens, the federal funds rate will be chosen such that it captures the different effects. Based on the problem outlined above, the optimal monetary policy rules, both under commitment and discretion, can be derived. This topic will be investigated in more details in another paper. Alternatively, in the following, by comparing the generated volatility and welfare losses from the optimal monetary policy and other commonly used policy rules, we can obtain the best policy in terms of the minimized welfare losses.

4.2.2 Evaluation of policy actions

This section considers the optimal policy derived above and a number of simple policy rules, and provides a quantitative evaluation of their impact on macroeconomic volatility and welfare. Given the parameters estimated in last section, the evaluation is based on the unconditional period losses. I will consider two policy rules. The first one is the Taylor rule used in this paper for the estimation. This rule disciplines the nominal interest rate to respond to the deviation of inflation and output gap from their counterparts obtained under the flexible wage and price economy subject to information friction. The second rule is the flexible inflation targeting rule. This rule is obtained when the coefficients for both output gap and the change of output gap become zero, whereas the coefficient for the inflation remains as estimated. Under this rule, the policy will change whenever inflation deviates, but will not when there is only variation in output gap.

Table 8 reports the volatility of macroeconomic variables and welfare losses under the optimal monetary policy, the Taylor rule, and the inflation targeting rule. For comparison, the welfare costs and volatility derived from both uncertainty and certainty are displayed. The welfare costs are the average periods welfare losses in terms of consumption units, as defined in last section, relative to the welfare losses implied by the optimal policy under uncertainty, which is set to zero. The volatility is characterized by the standard deviation of the output gap, price inflation and wage inflation.

Starting with the comparison between different policies under uncertainty at the left panel. The standard deviation of price inflation is 0.28 under the optimal policy, 114% lower than under the Taylor rule and 86% lower than under the inflation targeting rule. The volatility of wage inflation is the same under both the Taylor rule and inflation targeting rule, but 23% higher than that under the optimal policy. The volatility of output gap is highest under the inflation targeting rule, and lowest under the optimal policy. Not surprisingly, the welfare losses of both Taylor rule and inflation targeting rule are bigger than those under the optimal policy. The inflation targeting rule, however, generates a 17% welfare gain compared to the Taylor rule, possibly due to the decrease of the price inflation volatility. This result is consistent with the findings of Bernanke and Gertler (2001) who emphasizes the implications of uncertainty on the central bank's decisions and advocate the effectiveness of the inflation targeting rule. It implies that, when the central bank has difficulty to observe and differentiate the sources of inflation and output volatility, that is, whether due to aggregate supply or demand disturbances, the policy following inflation targeting rule could generate lower welfare losses compared to the widely conducted Taylor rule. When policy responds aggressively to an appropriate weight of price inflation, the inflation targeting rule will overperform the Taylor rule to a substantial extent with lower volatilities of price inflation, wage inflation and output gap, and therefore lower welfare losses. For example, when the coefficient for inflation doubles in the inflation targeting rule, the welfare losses will further decrease by more than 50%.

When considering the implications of different policies under certainty, the optimal policy still yields the lowest welfare losses, due to the lowest standard deviation of price inflation, wage inflation and output gap. Interestingly, given the parameters from the baseline estimation, the inflation targeting rule is worse than the Taylor rule, possibly due to the larger fluctuations in the output gap that result from following the rule. The result suggests that, when economic environment enables the public and central bank to observe the sources of variation, policy actions based on the Taylor rule gives higher welfare effects.

Finally, comparing across different environments, we have two interesting findings. First, the welfare gain of the optimal monetary policy relative to other simple rules is more under uncertainty than under certainty. This is likely due to the fact that information friction may serve as a propagation mechanism for markup shocks. As a result, the benefit brought by the optimal policy to stabilize marginal costs at a level consistent with firm's desired markup relative to other simple rules is bigger under uncertainty than under certainty. Second, in terms of economic volatility, the welfare losses of the optimal policy are slightly lower under uncertainty than under certainty, due to the lower aggregate volatility. However, the actual welfare could be bigger under certainty, when the welfare losses due to the information friction are considered. As stated earlier, information friction generates an inefficiently low level of output, which is not captured in the current measure of welfare. When these losses are accounted for, no matter following which policy actions, the welfare under certainty will be enhanced compared to the same action under uncertainty. Therefore it is important for the central bank and private agents to have access to more information so that the distortion resulting from information friction would be reduced to minimum.

4.2.3 Welfare effects of information

In the previous analysis, the central bank can affect economic dynamics through different policy actions. Economic agents, on the other hand, can observe policy actions and know their effects on macroeconomic variables, including the output that they extract information from. The policies, however, do not affect the learning structure, that is, how the updating of beliefs rely on different signals. Suppose now that the central bank can affect agents' learning and thus behavior by controlling the *precision* of information received by private agents. For example, it can decide whether to make some aggregate statistics available to private agents, which could affect market "sentiment" and thus the precision of the signal ϕ[13]. What are the welfare implications under this decision? Given the parameters as estimated in the benchmark framework, I examine the effects of variation in the precision of ϕ on social welfare under the Taylor rule. The precision is characterized by $\log(1/\sigma_v)$ where σ_v is the standard deviation of the "sentiment" shock. Intuitively, when σ_v increases, ϕ becomes less precise and informative.

Figure 9 shows the effect of the changes of $\log(1/\sigma_v)$ on welfare measured by W as derived in last section. The welfare is monotonically increasing in $\log(1/\sigma_v)$. The higher the precision of ϕ, the more closely agents' decisions rely on the underlying fundamentals. One thing to note is that agents' learning structure is influenced by σ_v. When the signal is less precise, the Kalman gain parameters representing the learning from ϕ decline and agents rely more on the signal of output to update beliefs. If, instead, the welfare is also dependent on the level of output, the increase of welfare will be more dramatic when the precision of information increases. The underlying argument is similar with Lorenzoni (2010): under information friction, the best achievable welfare corresponds to constrained efficient allocation, and more precise information will unambiguously increase output and therefore improve welfare.

When the same experiment is conducted under the optimal policy, better informational environment again leads to higher social welfare and lower aggregate volatility. It implies that, if the central bank aims to decrease aggregate volatility and increase social welfare, it is best to disclose available information to the public frequently and precisely so that the public can

[13] The notion that more precise information about aggregate variables has important welfare implications has been discussed in Hellwig (2005) and Lorenzoni (2010). In those two papers, more precise signals render prices' differentials more in line with productivity differentials across individuals and thus improves social welfare.

better learn about the underlying fundamentals and make decisions more close to the first best allocation. This implication mirrors the statement by Poole (1998) about the importance of knowing the economic state:" *My bottom line is that market participants should concentrate on the fundamentals. If the bond traders can get it right, they'll do most of the stabilization work for us, and we at the Fed can sit back and enjoy life."*

5 Conclusion

Monetary policy is conducted in an environment of substantial uncertainty. In this paper, I develop a general equilibrium DSGE framework augmented with information friction in terms of the indistinguishability between the supply-side TFP shock and demand-side IST shock and study its implication on business cycles and monetary policy. I find that the policy following the Taylor rule responding to inflation and output gap can still stabilize the economy under information friction. However, the existence of uncertainty could create constraints for the design of the optimal monetary policy relative to the certainty benchmark. Moreover, under uncertainty, the inflation targeting rule generates a higher welfare than the Taylor rule, but lower welfare under certainty.

This paper studies one type of uncertainty faced by the central bank that has not been fully investigated before. As Bernanke (2007) mentions, there are two other types of uncertainty which also matter for the decisions of the monetary authority, namely the uncertainty about the structure of the economy and the reaction of private agents to policies. The framework I develop in this paper can naturally be extended to study these two types of uncertainty and their implications on economic dynamics and monetary policy. On the other hand, the feedback feature of the optimal monetary policy is also intriguing. When the central bank tries to minimize welfare losses, it takes the economic environment as an input factor for the loss function. Different decisions express different understanding of the central bank on the current economic state, which in turn, could be learned by the public on observing the policy. Thus the public may behave not only based on its own expectation on the economic state, but also on the information extracted from the optimal policy. It would be interesting to analyze different effects of monetary policy on the economy when these additional learnings by monetary authority and/or private agents are considered, which allows a more rich informational and learning dynamics.

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Table 1. Prior densities and posterior estimates of structural parameters.

Param	Description	Prior			Posterior			
		Density	Mean	Std	Mode	Median	Std.	[5th,95th]
κ	I adjustment cost	Gamma	4	1.5	6.550	6.587	0.955	[5.84,7.30]
h	habit formation	Beta	0.7	0.1	0.805	0.784	0.039	[0.72,0.82]
θ_w	wage stickiness	Beta	0.5	0.1	0.766	0.778	0.039	[0.75,0.81]
θ_p	price stickiness	Beta	0.5	0.1	0.518	0.545	0.046	[0.50,0.58]
ι_w	wage indexation	Beta	0.5	0.1	0.539	0.563	0.106	[0.46,0.66]
ι_p	price indexation	Beta	0.5	0.1	0.174	0.160	0.076	[0.09,0.22]
Φ	share of fixed cost	Gamma	1.25	0.1	1.225	1.221	0.025	[1.15,1.27]
φ_R	Taylor rule smoothing	Beta	0.75	0.1	0.876	0.885	0.017	[0.87,0.90]
φ_π	Taylor rule inflation	Normal	1.5	0.3	1.869	1.845	0.164	[1.73,1.98]
φ_y	Taylor rule output	Normal	0.2	0.05	0.121	0.125	0.025	[0.11,0.14]
φ_{dy}	Taylor rule output gap	Normal	0.2	0.05	0.126	0.130	0.019	[0.11,0.16]
α	capital share	Beta	0.3	0.1	0.198	0.195	0.019	[0.18,0.21]
β	discount factor	Gamma	0.9	0.05	0.998	0.999	0.095	[0.99,1.01]
$P_{1,1}$	Kalman gain of z on y	N/A			0.818			
$P_{1,2}$	Kalman gain of z on ϕ	N/A			-0.017			
$P_{2,1}$	Kalman gain of k on y	N/A			0.016			
$P_{2,2}$	Kalman gain of k on ϕ	N/A			0.626			
$P_{3,1}$	Kalman gain of q on y	N/A			0.009			
$P_{3,2}$	Kalman gain of q on ϕ	N/A			0.415			

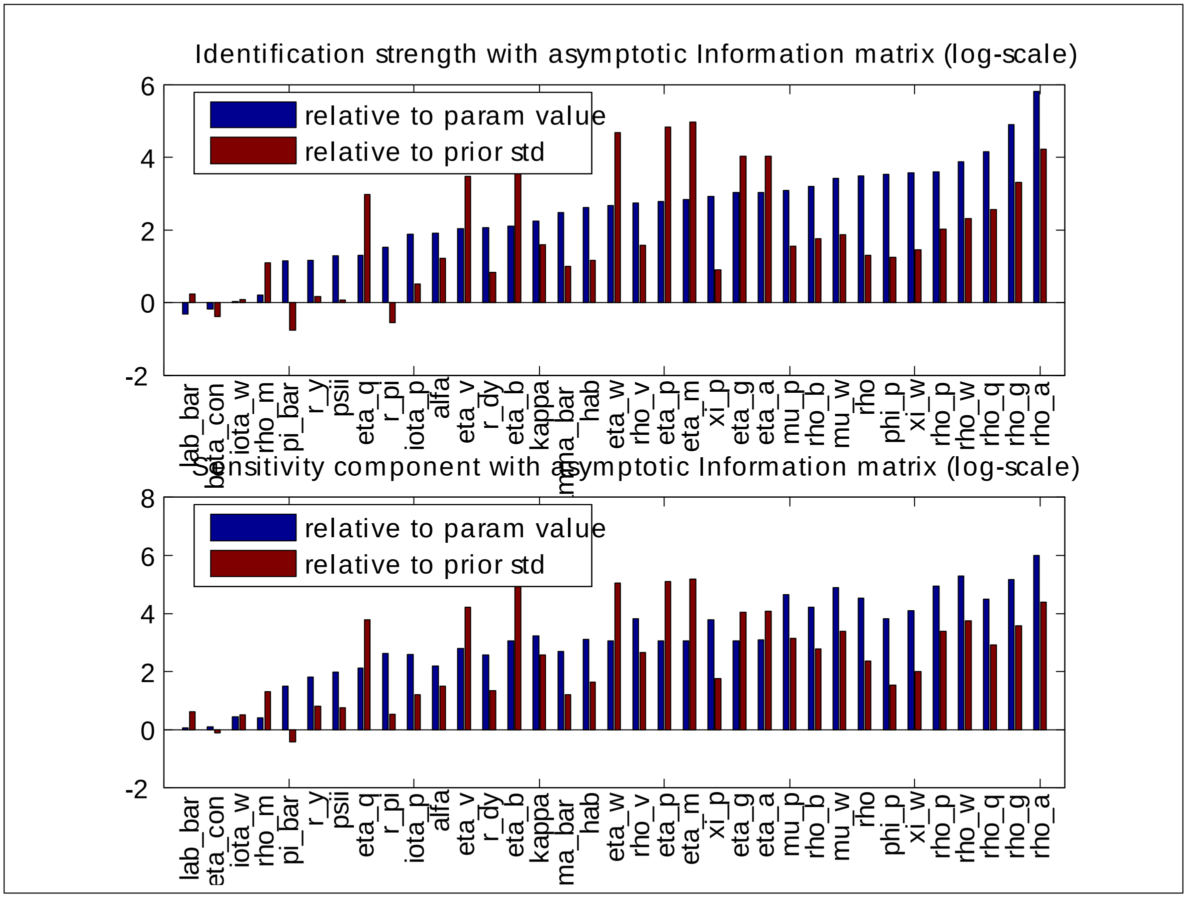
This table shows the prior densities and posterior estimates of the structural parameters from the estimation of the benchmark model with information friction. I use the Bayesian methodology that maximizes the log posterior likelihood and the Metropolis-Hastings Monte Carlo technique to obtain a complete posterior distribution of the parameters. The data used for the estimation spans the periods between 1969QIII and 2006QIV, including real GDP per capita, consumption, investment, hours worked, federal funds rate, inflation, and the forecasts of real GDP from the Surveys of Professional Forecasters.

Table 2. Prior densities and posterior estimates of stochastic shocks

Param	Description	Prior			Posterior			
		Density	Mean	Std	Mode	Median	Std.	[5th,95th]
ρ_z	TFP	Beta	0.5	0.1	0.983	0.986	0.004	[0.98,0.99]
ρ_b	asset premium	Beta	0.5	0.1	0.133	0.172	0.078	[0.11,0.24]
ρ_g	Gov spending	Beta	0.5	0.1	0.972	0.974	0.010	[0.96,0.99]
ρ_q	IST	Beta	0.5	0.1	0.617	0.629	0.054	[0.57,0.68]
ρ_m	monetary policy	Beta	0.5	0.1	0.201	0.208	0.064	[0.15,0.27]
ρ_p	price markup	Beta	0.5	0.1	0.986	0.983	0.009	[0.98,0.99]
ρ_w	wage markup	Beta	0.5	0.1	0.981	0.972	0.010	[0.95,0.98]
ρ_v	signal error	Beta	0.5	0.1	0.342	0.339	0.058	[0.30,0.40]
σ_z	TFP	InvG	0.1	1	0.491	0.484	0.025	[0.44,0.52]
σ_b	asset premium	InvG	0.1	1	0.359	0.369	0.030	[0.34,0.41]
σ_g	gov spending	InvG	0.1	1	0.738	0.753	0.035	[0.73,0.79]
σ_q	IST	InvG	0.1	1	0.567	0.561	0.055	[0.51,0.60]
σ_m	monetary policy	InvG	0.1	1	0.228	0.224	0.012	[0.21,0.24]
σ_p	price markup	InvG	0.1	1	0.217	0.219	0.023	[0.19,0.24]
σ_w	wage markup	InvG	0.1	1	0.269	0.278	0.018	[0.26,0.30]
σ_v	signal error	InvG	0.1	1	0.498	0.501	0.092	[0.48,0.52]
ϖ_p	MA in price	Beta	0.5	0.1	0.839	0.850	0.055	[0.79,0.89]
ϖ_w	MA in wage	Beta	0.5	0.1	0.933	0.926	0.021	[0.91,0.94]
log likelihood under uncertainty		-1533.66						
log likelihood under certainty		-1689.92						

This table shows the prior densities and posterior estimates of the shock parameters from the estimation of the benchmark model with information friction. I use the Bayesian methodology that maximizes the log posterior likelihood and the Metropolis-Hastings Monte Carlo technique to obtain a complete posterior distribution of the parameters. The data used for the estimation spans the periods between 1969QIII and 2006QIV, including real GDP per capita, consumption, investment, hours worked, federal funds rate, inflation, and the forecasts of real GDP from the Surveys of Professional Forecasters.

Table 3: Identification strength of parameters



This table shows the identification strength of the estimated parameters in the model. The model parameters on the x-axis are ranked in increasing order of strength of identification. I first perform the point estimation at the prior means and then loop over 250 replicas. See Ratto (2011) for the details of the algorithms for the identification.

Table 4: Standard deviations and correlations for observable variables in the baseline model

Variables	S.d. relative to GDP			Corr with GDP		
	Uncertainty	Certainty	Data	Uncertainty	Certainty	Data
Y	1.82	1.99	1.74	1	1	1
C	0.89	0.78	0.84	0.51	0.65	0.86
I	3.09	4.09	3.37	0.82	0.92	0.90
L	0.86	0.87	0.93	0.86	0.94	0.88
R	0.38	0.36	0.49	0.93	0.95	0.96
π	0.34	0.35	0.39	0.72	0.69	0.75

The left panel of the table reports the standard deviation of the six observable variables relative to that of real GDP per capita, for both the baseline model and the model without information friction. The right panel reports the correlation of the six observable variables with real GDP per capita, for both the baseline model and the model without information friction. Given parameters from the estimation of the baseline model as in Table 1 and 2, the model statistics are computed based on the HP-filtered data from the simulation of the modeled economy of 2100 periods. The data statistics are calculated from the data sample that is used for the estimation. The data statistics are based on logged and HP-filtered data with smoothing parameter of 1600.

Table 5: Marginal likelihood of various VAR models and DSGE models

Various models	Likelihood values
VAR(1)	-1719.83
VAR(2)	-1764.98
VAR(3)	-1842.12
VAR(4)	-1988.37
DSGE under certainty	-1689.92
DSGE under uncertainty	-1533.66

This table compares the marginal likelihood of the baseline DSGE model, the DSGE model under certainty and the unconstrained VAR models with different lags. All have the same vector of observable variables as is used for the baseline estimation, and are estimated over the sample period between 1969QIII and 2006QIV.

Table 6: Predicted mean squared errors for selected variables by VAR(1) and DSGE models

	GDP	Consumption	Investment	Hours	Interest rate	Inflation
VAR(1)	Mean squared errors					
1Q	0.70	0.81	1.98	0.65	0.11	0.29
4Q	1.85	1.76	6.03	1.94	0.42	0.31
8Q	2.67	2.54	9.11	2.32	0.69	0.51
DSGE certainty	Percentage gains (+) or losses (-)					
1Q	6.02	18.64	15.65	1.53	-6.95	0.68
4Q	18.17	23.65	23.98	11.85	0.38	28.65
8Q	21.47	21.71	27.39	19.75	16.96	34.87
DSGE uncertainty	Percentage gains (+) or losses (-)					
1Q	7.12	20.78	10.01	2.31	-7.85	2.98
4Q	19.13	22.95	21.65	10.85	-3.64	36.98
8Q	22.78	20.64	27.74	21.23	5.78	58.74

Table 3 reports the pseudo out-of-sample forecast performance in terms of mean squared errors for different horizons based on, the VAR and DSGE model models. These models are estimated over the sample from 1969QIII to 1989QIV. Based on the estimation, the models are used to forecast the seven observable data series from 1990QI to 2006QIV. The mean squared errors are then computed as the square root of the mean of the squared difference between the forecasted and corresponding observed values over the sample.

Table 7: Variance decomposition at business cycle frequencies (in percentage)

Variables/ Shocks	Neutral Productivity	Equity premium	Government spending	Investment technology	Monetary policy	Price mark-up	Wage mark-up	Sentiment
Y	24.97	15.82	22.44	8.08	9.41	3.94	13.97	3.37
C	19.20	33.70	15.04	2.19	8.45	3.19	13.09	5.13
I	24.21	6.43	17.21	21.45	4.71	3.29	6.81	7.90
n	22.26	16.56	23.08	5.23	9.10	3.33	15.16	4.87
$E(q)$	0.10	0.00	0.00	43..02	0.00	0.00	0.00	56.88
$E(z)$	92.69	0.00	0.00	1.68	0.00	0.00	0.00	5.63

This table provides the decomposition of the effects of various shocks on included variables for an infinite horizon. Each number represents the contribution of a particular shock to the variable relative to the sum of the contribution of each shock to that variable.

Table 8: Evaluation of different policy actions

	Uncertainty			Certainty		
	Optimal policy	Taylor rule	Inflation targeting rule	Optimal policy	Taylor rule	Inflation targeting rule
$\sigma(\tilde{Y})$	1.31	1.44	1.78	1.50	1.54	1.98
$\sigma(\pi^p)$	0.28	0.60	0.52	0.31	0.61	0.57
$\sigma(\pi^w)$	0.39	0.48	0.48	0.42	0.56	0.58
L	0	-1.15	-0.96	-0.26	-1.09	-1.32

The first three rows of this table show the volatilities of output gap, price inflation and wage inflation for the baseline uncertainty model and the certainty model under the optimal policy, the Taylor rule and the inflation targeting rule respectively. The results are computed based on the HP-filtered data from the simulation of the baseline economy and the certainty economy of 2100 periods, using the parameters estimated from the baseline model. The last row shows the average periods welfare losses under different policy actions. The welfare losses for the baseline economy generated by the optimal policy is normalized to 0. The welfare losses for the other policies are computed relative to 0

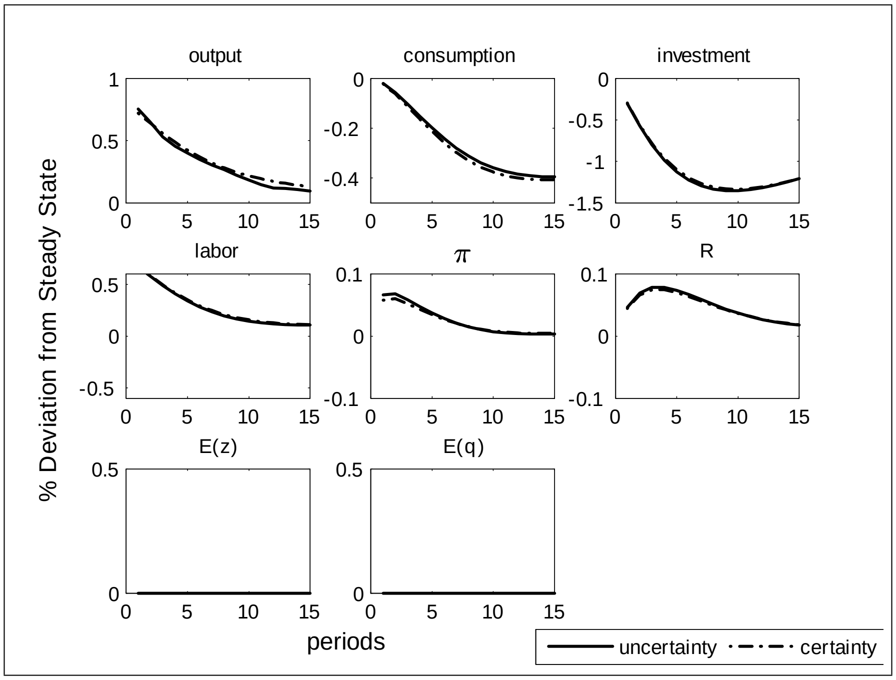


Figure 3: Effects of government spending shock

The results show the responses of macroeconomic variables to a one standard deviation of government spending shock. The economy is assumed to start at a correct steady state initially, such that all the variables are at their steady state level, and the expectations about the unobserved fundamentals coincide with their actual values. The responses for any period in the figure show the economic activities after the observation of the current shock, represented as the percentage deviations from the steady state.

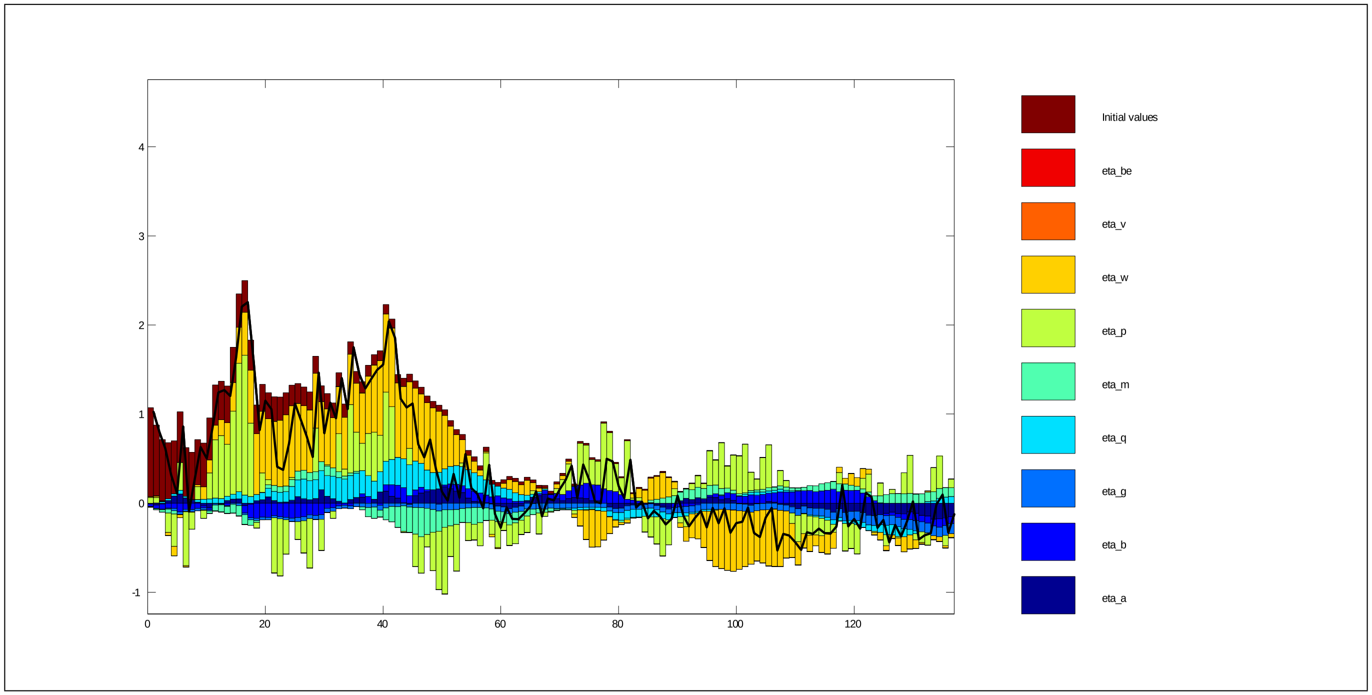


Figure 1: Historical variance decomposition for inflation

The historical variance decomposition of inflation is computed by dividing the inflation into its sub-components with respect to the estimated structural shocks. Each color represents the contribution of one particular shock to the movement of inflation at any period.

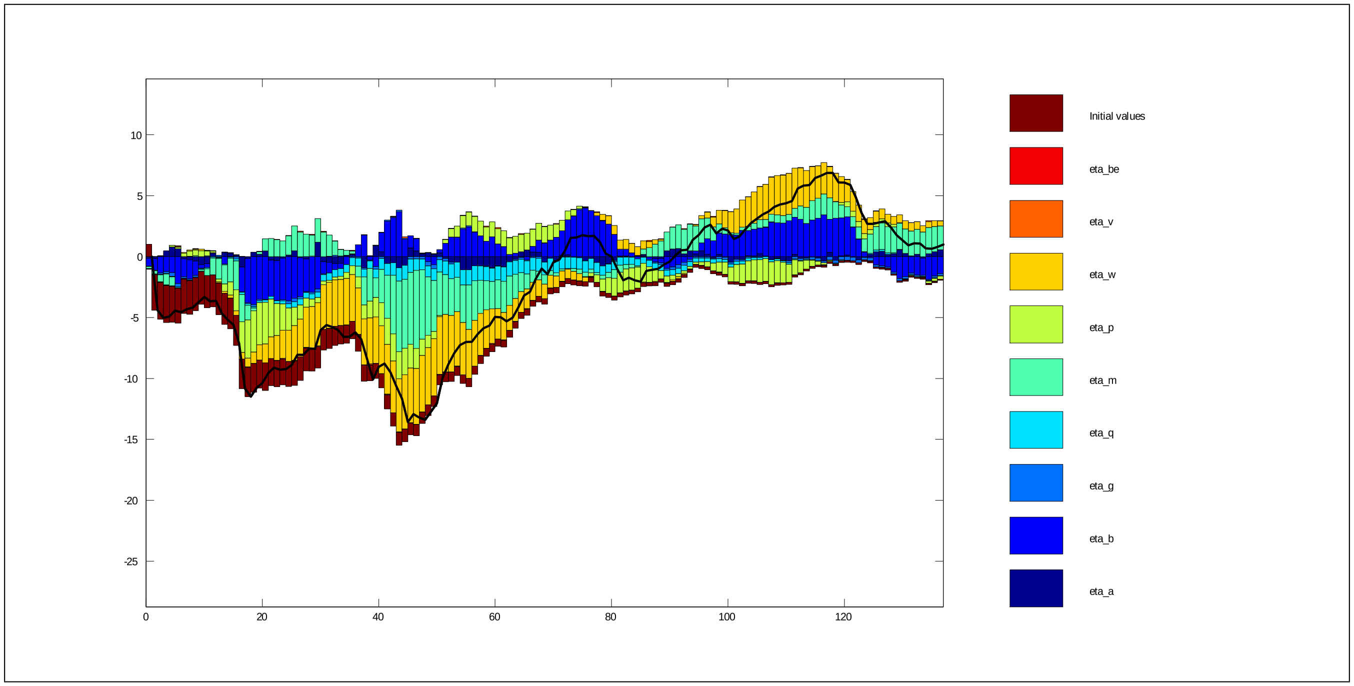


Figure 2: Historical variance decomposition for output gap

The historical variance decomposition of inflation is computed by dividing the inflation into its sub-components with respect to the estimated structural shocks. Each color represents the contribution of one particular shock to the movement of output gap at any period.

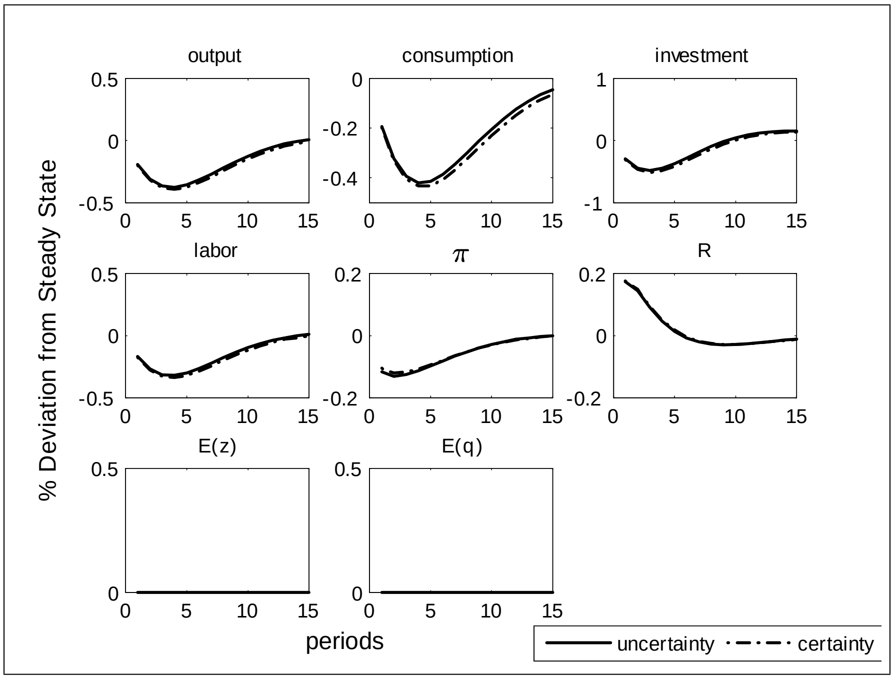


Figure 4: Effects of monetary policy shock

The results show the responses of macroeconomic variables to a one standard deviation of monetary policy shock. The economy is assumed to start at a correct steady state initially, such that all the variables are at their steady state level, and the expectations about the unobserved fundamentals coincide with their actual values. The responses for any period in the figure show the economic activities after the observation of the current shock, represented as the percentage deviations from the steady state.

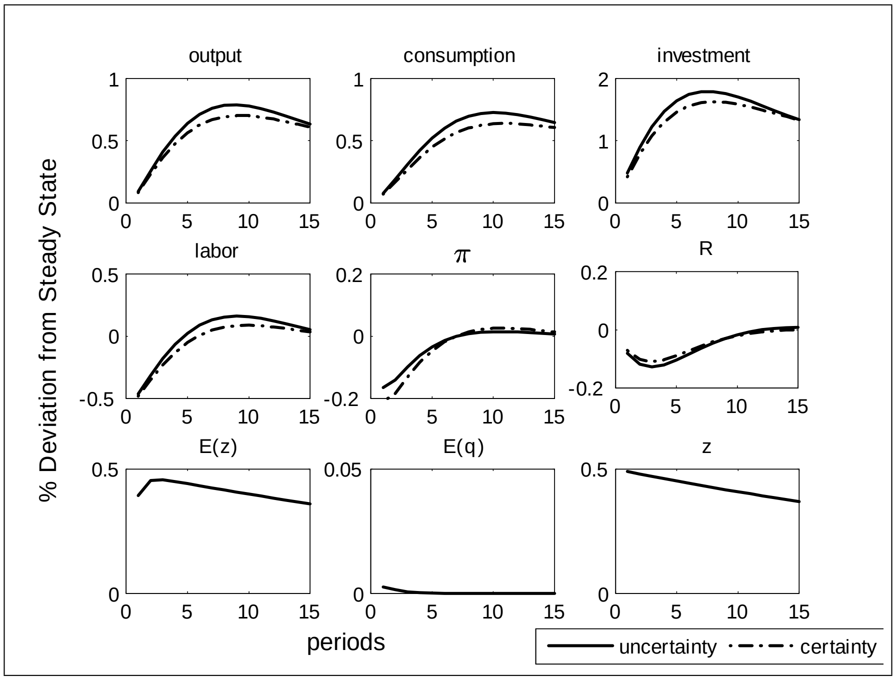


Figure 5: Effects of TFP shock

The results show the responses of macroeconomic variables to a one standard deviation of the TFP shock. The economy is assumed to start at a correct steady state initially, such that all the variables are at their steady state level, and the expectations about the unobserved fundamentals coincide with their actual values. The responses for any period in the figure show the economic activities since the observation of the current period signals and till the observation of the next period signals, represented as the percentage deviations from the steady state.

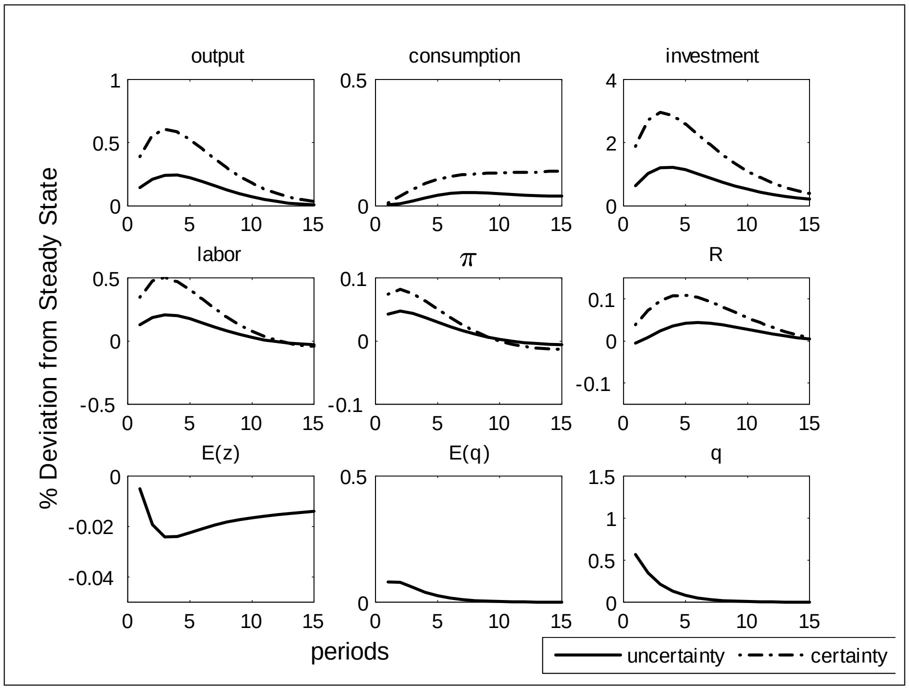


Figure 6: Effects of IST shock

The results show the responses of macroeconomic variables to a one standard deviation of the IST shock. The economy is assumed to start at a correct steady state initially, such that all the variables are at their steady state level, and the expectations about the unobserved fundamentals coincide with their actual values. The responses for any period in the figure show the economic activities since the observation of the current period signals and till the observation of the next period signals, represented as the percentage deviations from the steady state.

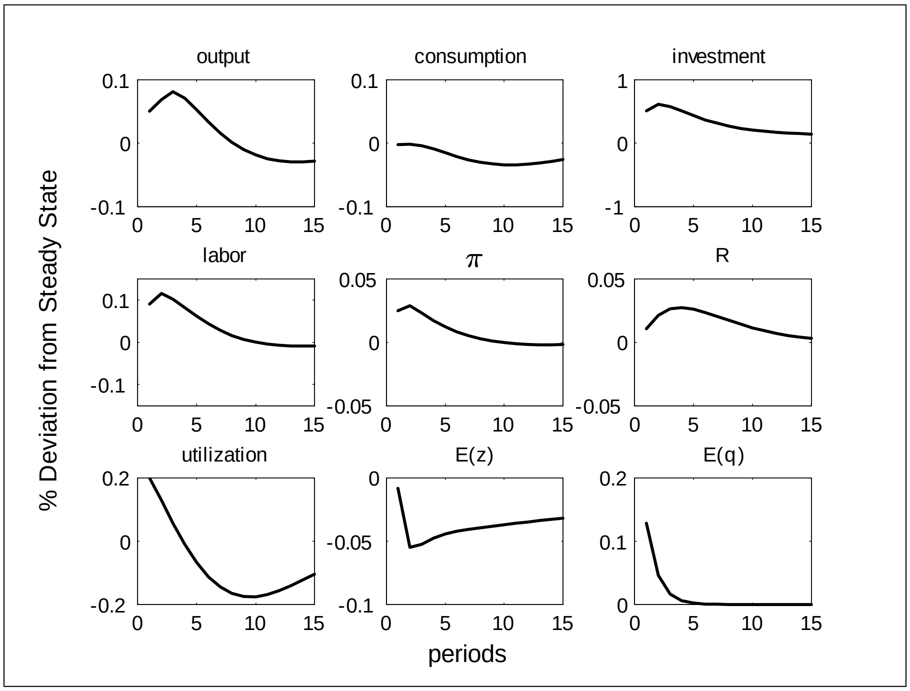


Figure 7: Effects of sentiment shock

The results show the responses of macroeconomic variables to a one standard deviation of the sentiment shock. The economy is assumed to start at a correct steady state initially, such that all the variables are at their steady state level, and the expectations about the unobserved fundamentals coincide with their actual values. The responses for any period in the figure show the economic activities since the observation of the current period signals and till the observation of the next period signals, represented as the percentage deviations from the steady state.

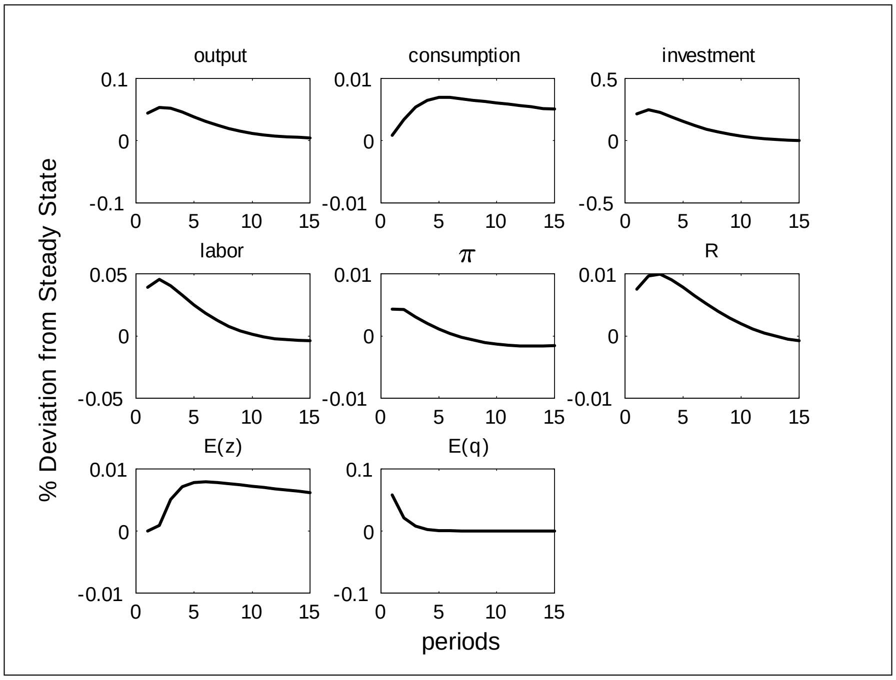


Figure 8: Effects of incorrect beliefs on IST

The results show the responses of macroeconomic variables to an initial incorrect belief about IST, the level of which is equal to the immediate response of IST to a two standard deviation IST shock. Initially, the beliefs about the other unobserved fundamentals (TFP and effective capital) are consistent with their actual values. There are no other shocks happening subsequently.

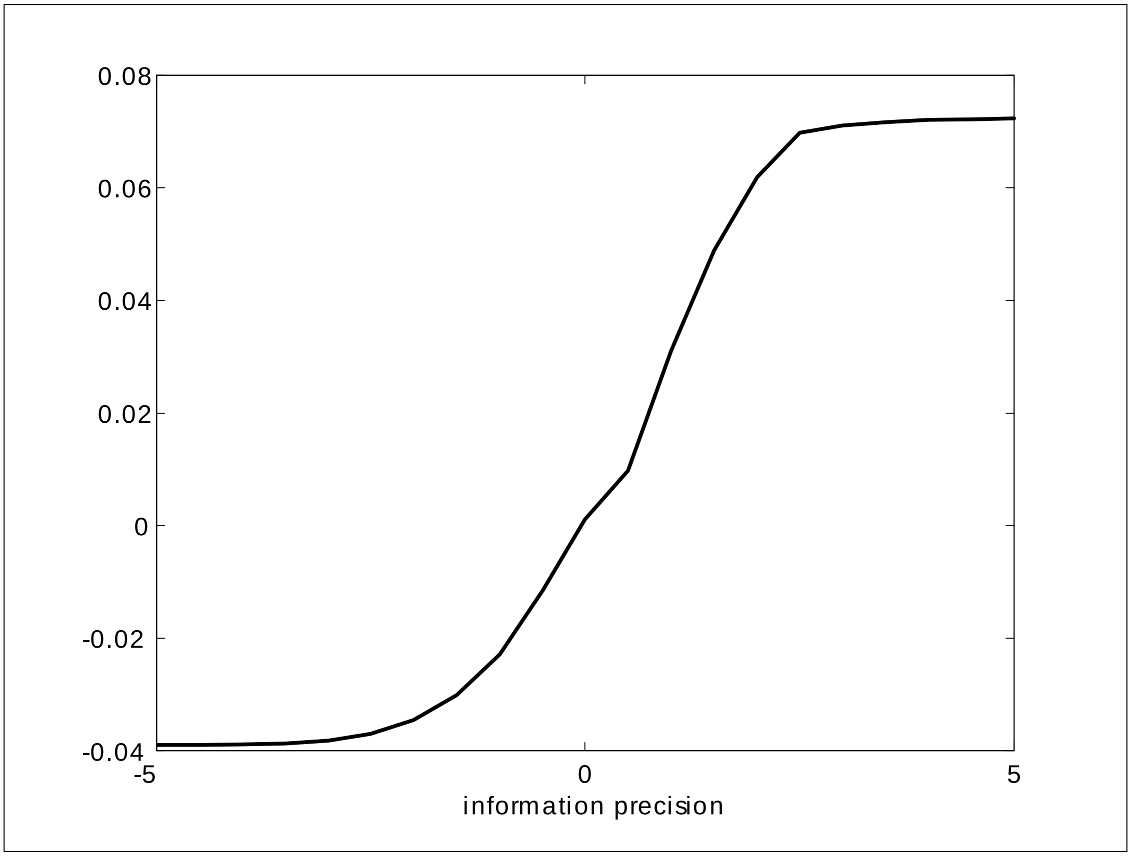


Figure 9. Welfare effects of information precision

The horizontal axis is $\log(1/\sigma_v)$, where σ_v is the standard deviation of the sentiment shock. The higher the standard deviation, the less precise the informative signals agents can observe. The vertical axis is the measure of welfare expressed in terms of differences from their value at $\sigma_v = 1$.

7 Appendix

7.1 Appendix 1: Stylied facts

The US tech boom-bust cycle around 2000s has been widely attributed to the over optimasm about the progress of investment technology, which is strongly related to information and communication technology. The boom started with optimistic expectation about the unobserved progress of IST compared to TFP and ended with downward revision of the expectations and of investment and output. Figure 0 depicts some stylized facts indicative of this episode. During the 2000s recession, the volatility of the NASDAQ index is higher than the Willshire index. Comparing across time, the decline of the NASDAQ preceded the 2000s recession and was more dramatic than during the Great Recession.

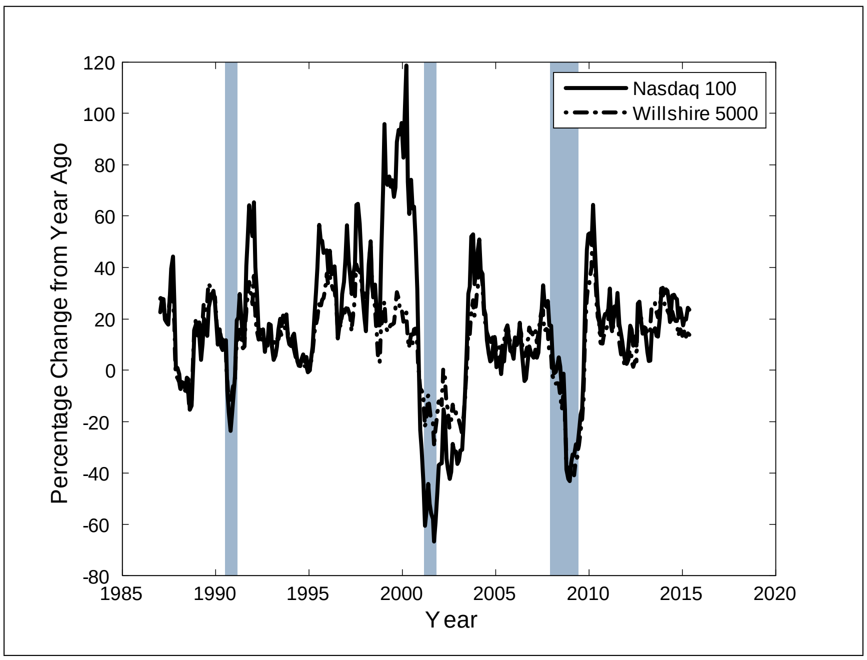


Figure 0. Performance of NASDAQ and WILLSHIRE indices

The grey bar shows the NBER recession periods. The lines show the percentage change from one year ago. NASDAQ 100 measures the expected performance of high-tech and innovation firms. Willshire 5000 measures the expected performance of the top 5000 big companies covering all sectors .

7.2 Appendix 2: Evolution of beliefs

For the TFP and IST shocks, given their posterior beliefs, and their known evolution as AR(1) processes:

$$z_{t+1} = \rho_z z_t + \varepsilon_{t+1}^z$$

$$q_{t+1} = \rho_q q_t + \varepsilon_{t+1}^q$$

It can be derived the prior belief about TFP for period $t+1$, g_{t+1}^Z, and the prior belief about IST for period $t+1, g_{t+1}^Q$ by conjugate distribution calculation as:

$$z_{t+1}\tilde{}N(\rho_z \mu_z, \rho_z^2 \sigma_z^2 + \sigma_{\varepsilon z}^2)$$

$$q_{t+1}\tilde{}N(\rho_q \mu_q, \rho_q^2 \sigma_q^2 + \sigma_{\epsilon q}^2)$$

For the effective capital, given the posterior belief, and the known evolution:

$$k_{t+1} = (1-\delta)k_t + e^{q_t}\left(1 - S\left(\frac{Y_t^I}{Y_{t-1}^I}\right)\right)Y_t^I$$

The cumulative density function for a random variable $K_{t+1} = (1-\delta)K_t + e^{q_t}\left(1 - S\left(\frac{Y_t^I}{Y_{t-1}^I}\right)\right)Y_t^I \leq k'$ can be written as

$$\begin{aligned}
F_{t+1}^K(k'|m_t^K, m_t^q) &= Prob\ ((1-\delta)K_t + e^{Q_t}\left(1 - S\left(\frac{Y_t^I}{Y_{t-1}^I}\right)\right)Y_t^I \leq k') \\
&= \int\int_{k(1-\delta)+e^{q_t}\left(1-S\left(\frac{Y_t^I}{Y_{t-1}^I}\right)\right)Y_t^I \leq k'} m_t^K m_t^q dkdq \\
&\quad \text{where } k \text{ is the realized value of } K_t, q \text{ is the realized value of } Q_t \\
&= \int_{q=0}^{\infty}\int_{k=0}^{\frac{k'-e^{q_t}\left(1-S\left(\frac{Y_t^I}{Y_{t-1}^I}\right)\right)Y_t^I}{(1-\delta)}} m_t^K m_t^Q dkdq \\
&= \int_{k=0}^{\infty}\int_{q=0}^{\log\left(\frac{k'-k(1-\delta)}{\left(1-S\left(\frac{Y_t^I}{Y_{t-1}^I}\right)\right)Y_t^I}\right)} m_t^K m_t^Q dkdq
\end{aligned}$$

and therefore the prior belief about the effective capital for period $t+1$, g_{t+1}^K is

$$
\begin{aligned}
g^K_{t+1}(k') &= \int_{k=0}^{\infty} \frac{1}{k'-k(1-\delta)} m^K_t m^Q_t \left(\log(\frac{k'-k(1-\delta)}{\left(1-S\left(\frac{Y^I_t}{Y^I_{t-1}}\right)\right) Y^I_t}) \right) dk \\
&= \frac{1}{(1-\delta)} \int_{q=0}^{\infty} m^K_t \left(\frac{k' - e^q \left(1-S\left(\frac{Y^I_t}{Y^I_{t-1}}\right)\right) Y^I_t}{(1-\delta)} \right) m^Q_t dq
\end{aligned}
$$

Given this, the expected output for the next period can be derived as

$$
\begin{aligned}
F^Y_{t+1}\left(y' | g^Z_{t+1}, g^K_{t+1}, n', u'\right) &= Prob\ (e^{Z_{t+1}} (u_t K_{t+1})^{\alpha} {n_{t+1}}^{1-\alpha} \leq y') \\
&= \int\int_{e^{z'}(u'k')^{\alpha} n'^{1-\alpha} \leq y'} g^K_{t+1} g^Z_{t+1} dk' dz' \\
&= \int_{z'=0}^{\infty} \int_{k'=0}^{\left(\frac{y'}{e^{z'} u'^{\alpha} n'^{1-\alpha}}\right)^{1/\alpha}} g^K_{t+1} g^Z_{t+1} dk' dz' \\
&= \int_{k'=0}^{\infty} \int_{z'=0}^{\log(\frac{y'}{u'^{\alpha} k'^{\alpha} n'^{1-\alpha}})} g^K_{t+1} g^Z_{t+1} dk' dz'
\end{aligned}
$$

and thus the prior belief about output in period $t+1$ is

$$
\begin{aligned}
g^Y_{t+1}(y') &= \frac{1}{\alpha} y'^{\frac{1}{\alpha}-1} n'^{\frac{\alpha-1}{\alpha}} \int_{z'=0}^{\infty} \left(\frac{1}{e^{z'}}\right)^{1/\alpha} g^K_{t+1} \left(\left[\frac{y'}{e^{z'} u'^{\alpha} n'^{1-\alpha}} \right]^{1/\alpha} \right) g^Z_{t+1} dz' \\
&= \int_{k'=0}^{\infty} \frac{u'^{\alpha} k'^{\alpha} n'^{1-\alpha}}{y'} g^K_{t+1} g^Z_{t+1} \left(\log\left(\frac{y'}{u'^{\alpha} k'^{\alpha} n'^{1-\alpha}} \right) \right) dk'
\end{aligned}
$$

7.3 Appendix 3: Evolution of linearized beliefs

The notation for this appendix is:

1. M^K_t, M^Z_t, M^Q_t are respectively the prior means of capital, TFP and IST respectively for period t, given at the beginning of period t; $\Sigma^K_t, \Sigma^Z_t, \Sigma^Q_t$ are respectively the prior covariances of capital, TFP and IST respectively for period t.

2. $\mu^K_t, \mu^Z_t, \mu^Q_t$ are respectively the posterior means of capital, TFP and IST at period t; $\sigma^K_t, \sigma^Z_t, \sigma^Q_t$ are respectively the posterior variances of capital, TFP and IST.

3. If X_t is the true value of the variable X, $E_{t-1}X_t$ denotes prior belief of the variable X, and $E_t X_t$ the posterior belief. So for example $M^Z_t = E_{t-1} z_t$ and $\mu^Z_t = E_t z_t$. Similar expression holds for q and $\tilde{k}$.

4. For any variable X_t, define $X_t \equiv \bar{X} e^{\tilde{X}_t}$, so $\tilde{X}_t = \log X_t - \log \bar{X}$, and $\bar{X}$ is the steady state of variable X. For small deviations, $\bar{X} e^{\tilde{X}_t} \approx \bar{X}(1+\tilde{X}_t)$, which enables the linearization of the system.

Then we can rewrite the relevant functions as linear equations:

$$\frac{\tilde{Y}_t}{\Phi_p} = z_t + \alpha \tilde{k}_t + \alpha u_t + (1-\alpha)\tilde{n}_t$$

$$\tilde{\phi}_t = \rho_\phi \tilde{\phi}_{t-1} + (1-\rho_\phi) q_t + v_t$$

$$\tilde{k}_t = (1-\delta)\tilde{k}_{t-1} + \delta \tilde{Y}^I_{t-1} + \delta \kappa q_t$$

Together with the stochastic processes of TFP and IST shocks

$$\begin{aligned} z_{t+1} &= \rho_z z_t + \varepsilon^z_{t+1}, \varepsilon^z_{t+1} \sim N(\mu_z, \sigma^2_{\varepsilon_z}) \\ q_{t+1} &= \rho_q q_t + \varepsilon^q_{t+1}, \varepsilon^q_{t+1} \sim N(\mu_q, \sigma^2_{\varepsilon_q}) \end{aligned}$$

We now have a discrete time, linear and time varying state space system, where the measurement equations are:

$$\begin{pmatrix} \frac{\tilde{Y}_t}{\Phi_p} - (1-\alpha)\tilde{n}_t - \alpha u_t \\ \tilde{\phi}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & \alpha \\ 0 & 1-\rho_\phi & 0 \end{pmatrix} \cdot \begin{pmatrix} z_t \\ q_t \\ \tilde{k}_t \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \rho_\phi & 0 \end{pmatrix} \cdot \begin{pmatrix} \tilde{\phi}_{t-1} \\ \tilde{Y}^I_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ v_t \end{pmatrix}$$

and the state equations are:

$$\begin{pmatrix} z_t \\ q_t \\ \tilde{k}_t \end{pmatrix} = \begin{pmatrix} \rho_z & 0 & 0 \\ 0 & \rho_q & 0 \\ 0 & \delta\kappa & 1-\delta \end{pmatrix} \cdot \begin{pmatrix} z_{t-1} \\ q_{t-1} \\ \tilde{k}_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \delta \end{pmatrix} \cdot \begin{pmatrix} \tilde{\phi}_{t-1} \\ \tilde{Y}^I_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon^z_{t+1} \\ \varepsilon^q_{t+1} \\ 0 \end{pmatrix}$$

In order to obtain a Gauss-Markov process, it is assumed that the initial values of the random variables satisfy:

i) The initial TFP shock z_{-1} is a random Gaussian variable, independent of the noise processes, with $z_{-1} \sim N(M^Z_0, \Sigma^Z_0)$

ii) The initial IST shock q_{-1} is a random Gaussian variable, independent of the noise processes, with $q_{-1} \sim N(M^Q_0, \Sigma^Q_0)$

iii) The initial capital $\tilde{k}_{-1}$ is a random Gaussian variable, independent of the noise processes, with $\tilde{k}_{-1} \sim N(M^{\tilde{K}}_0, \Sigma^{\tilde{K}}_0)$

Then, the calculation follows:

Measurement update: Since $\tilde{y}_t/\Phi_p = z_t + \alpha\tilde{k}_t + \alpha\tilde{u}_t + (1-\alpha)\tilde{n}_t$, $\tilde{\phi}_t = \rho_\phi\tilde{\phi}_{t-1} + (1-\rho_\phi)q_t + v_t$, the conditional vector

$$\left(\begin{array}{c} z_t \\ \tilde{k}_t \\ q_t \\ \frac{\tilde{Y}_t}{\Phi_p} - (1-\alpha)\tilde{n}_t - \alpha\tilde{u}_t \\ \tilde{\phi}_t - \rho_\phi\tilde{\phi}_{t-1} \end{array} \right) | \mathbf{Y}_{t-1}, \mathbf{\Phi}_{t-1}, \tilde{n}_t, \tilde{u}_t$$

is Gaussian, with mean and variance:

$$\left[\begin{array}{c} \begin{pmatrix} M_t^Z \\ M_t^{\tilde{K}} \\ M_t^Q \end{pmatrix} \\ \begin{pmatrix} M_t^Z + \alpha M_t^{\tilde{K}} \\ M_t^Q \end{pmatrix} \end{array} \right],$$

$$\left[\begin{array}{cc} \begin{pmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{pmatrix}, & \begin{pmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \alpha & 0 \\ 0 & 1-\rho_\phi \end{pmatrix} \\ \begin{pmatrix} 1 & \alpha & 0 \\ 0 & 0 & 1-\rho_\phi \end{pmatrix} \begin{pmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{pmatrix}, & \begin{pmatrix} 1 & \alpha & 0 \\ 0 & 0 & 1-\rho_\phi \end{pmatrix} \begin{pmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \alpha & 0 \\ 0 & 1-\rho_\phi \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \sigma_v \end{pmatrix} \end{array} \right]$$

To compute $\left(\begin{array}{c} z_t \\ \tilde{k}_t \\ q_t \end{array} |Y_t, \Phi_t, \tilde{n}_t, \tilde{u}_t \right) = \left(\begin{array}{c} z_t \\ \tilde{k}_t \\ q_t \end{array} |y_t, Y_{t-1}, \phi_t, \Phi_{t-1}, \tilde{n}_t, \tilde{u}_t \right)$, which is the posterior belief on z_t, $\tilde{k}_t$ and q_t, we apply the formula for conditional expectation of Gaussian random variables, with everything preconditioned on Y_t and Φ_t. It follows that $\left(\begin{array}{c} z_t \\ \tilde{k}_t \\ q_t \end{array} |\mathbf{Y}_t, \mathbf{\Phi}_t, \tilde{n}_t, \tilde{u}_t \right)$ is Gaussian, with mean

$$\begin{pmatrix} \mu_t^Z \\ \mu_t^{\tilde{K}} \\ \mu_t^Q \end{pmatrix} = E\left(\begin{bmatrix} z_t \\ \tilde{k}_t \\ q_t \end{bmatrix} |Y_t, \Phi_t, \tilde{n}_t, \tilde{u}_t\right)$$

$$= \begin{bmatrix} M_t^Z \\ M_t^{\tilde{K}} \\ M_t^Q \end{bmatrix} + \begin{bmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{bmatrix} \begin{bmatrix} 1-\rho_\phi & 0 \\ \alpha & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left[\begin{pmatrix} 1 & \alpha & 0 \\ 0 & 0 & 1-\rho_\phi \end{pmatrix} \begin{pmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \alpha & 0 \\ 0 & 1-\rho_\phi \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \sigma_v \end{pmatrix}\right]^{-1}$$

$$\begin{bmatrix} \frac{\tilde{Y}_t}{\Phi_p} - (1-\alpha)\tilde{n}_t - \alpha\tilde{u}_t - M_t^Z - \alpha M_t^{\tilde{K}} \\ \tilde{\phi}_t - M_t^Q - \rho_\phi \tilde{\phi}_{t-1} \end{bmatrix}$$

$$= \begin{bmatrix} M_t^Z \\ M_t^{\tilde{K}} \\ M_t^Q \end{bmatrix} + P_t \begin{bmatrix} \tilde{y}_t/\Phi_p - (1-\alpha)\tilde{n}_t - -\alpha\tilde{u}_t - M_t^Z - \alpha M_t^{\tilde{K}} \\ \tilde{\phi}_t - M_t^Q - \rho_\phi \tilde{\phi}_{t-1} \end{bmatrix}$$

and covariance

$$\begin{pmatrix} \sigma_t^Z & \sigma_t^{\tilde{K},Z} & \sigma_t^{Z,Q} \\ \sigma_t^{\tilde{K},Z} & \sigma_t^{\tilde{K}} & \sigma_t^{\tilde{K},Q} \\ \sigma_{t,}^{Z,Q} & \sigma_t^{\tilde{K},Q} & \sigma_t^Q \end{pmatrix}$$

$$= cov\left(\begin{bmatrix} z_t \\ \tilde{k}_t \\ q_t \end{bmatrix} |Y_t, \Phi_t, \tilde{n}_t, \tilde{u}_t\right)$$

$$= \begin{bmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{bmatrix} - \begin{bmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \alpha & 0 \\ 0 & 1-\rho_\phi \end{bmatrix}$$

$$\left[\begin{pmatrix} 1 & \alpha & 0 \\ 0 & 0 & 1-\rho_\phi \end{pmatrix} \begin{pmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \alpha & 0 \\ 0 & 1-\rho_\phi \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \sigma_v \end{pmatrix}\right]^{-1}$$

$$\begin{bmatrix} 1 & \alpha & 0 \\ 0 & 0 & 1-\rho_\phi \end{bmatrix} \begin{bmatrix} \Sigma_t^Z & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^Q \end{bmatrix}$$

and denote the Kalman gain P as

$$
\begin{aligned}
P_t &= \begin{bmatrix} \Sigma_t^{Z} & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^{Q} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \alpha & 0 \\ 0 & 1-\rho_\phi \end{bmatrix} \times \\
& \left[\begin{pmatrix} 1 & \alpha & 0 \\ 0 & 0 & 1-\rho_\phi \end{pmatrix} \begin{pmatrix} \Sigma_t^{Z} & \Sigma_t^{\tilde{K},Z} & \Sigma_t^{Z,Q} \\ \Sigma_t^{\tilde{K},Z} & \Sigma_t^{\tilde{K}} & \Sigma_t^{\tilde{K},Q} \\ \Sigma_{t,t}^{Z,Q} & \Sigma_t^{\tilde{K},Q} & \Sigma_t^{Q} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \alpha & 0 \\ 0 & 1-\rho_\phi \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \sigma_v \end{pmatrix} \right]^{-1}
\end{aligned}
$$

Time update: Given the evolution processes for z,q and k, and that z_{t+1} and ε_{t+1}^{z} are independent, and q_{t+1} and ε_{t+1}^{q} are independent given signals. the posterior beliefs of the means are

$$
\begin{bmatrix} M_{t+1}^{Z} \\ M_{t+1}^{\tilde{K}} \\ M_{t+1}^{Q} \end{bmatrix} = E\left(\begin{bmatrix} z_{t+1} \\ \tilde{k}_{t+1} \\ q_{t+1} \end{bmatrix} |Y_t, \Phi_t, \tilde{n}_t, \tilde{u}_t \right) = \begin{pmatrix} \rho_z & 0 & 0 \\ 0 & 1-\delta & \delta\kappa \\ 0 & 0 & \rho_q \end{pmatrix} \begin{bmatrix} \mu_t^{Z} \\ \mu_t^{\tilde{K}} \\ \mu_t^{Q} \end{bmatrix} + \begin{bmatrix} 0 \\ \delta \\ 0 \end{bmatrix} \tilde{Y}_{t-1}^{I}
$$

and the posterior beliefs of the covariances are

$$
\begin{aligned}
& \begin{bmatrix} \Sigma_{t+1}^{Z} & \Sigma_{t+1}^{\tilde{K},Z} & \Sigma_{t+1}^{Z,Q} \\ \Sigma_{t+1}^{\tilde{K},Z} & \Sigma_{t+1}^{\tilde{K}} & \Sigma_{t+1}^{\tilde{K},Q} \\ \Sigma_{t+1}^{Z,Q} & \Sigma_{t+1}^{\tilde{K},Q} & \Sigma_{t+1}^{Q} \end{bmatrix} \\
= \; & cov\left(\begin{bmatrix} z_{t+1} \\ \tilde{k}_{t+1} \\ q_{t+1} \end{bmatrix} |\mathbf{Y}_t, \mathbf{\Phi}_t, \tilde{n}_t, \tilde{u}_t \right) \\
= \; & \begin{bmatrix} \rho_z & 0 & 0 \\ 0 & 1-\delta & \delta\kappa \\ 0 & 0 & \rho_q \end{bmatrix} \begin{bmatrix} \sigma_t^{Z} & \sigma_t^{\tilde{K},Z} & \sigma_t^{Z,Q} \\ \sigma_t^{\tilde{K},Z} & \sigma_t^{\tilde{K}} & \sigma_t^{\tilde{K},Q} \\ \sigma_{t,}^{Z,Q} & \sigma_t^{\tilde{K},Q} & \sigma_t^{Q} \end{bmatrix} \begin{bmatrix} \rho_z & 0 & 0 \\ 0 & 1-\delta & 0 \\ 0 & \delta\kappa & \rho_q \end{bmatrix} + \begin{bmatrix} \sigma_\varepsilon^{z} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_\varepsilon^{q} \end{bmatrix}
\end{aligned}
$$

Note that the conditional covariances do not depend on the measurement $\frac{\tilde{Y}_t}{\Phi_p}$ and $\tilde{\phi}_t$. The steady state of them can therefore be computed in advance, given the noise variances and model parameters. Thus the Kalman gain P will also be constant in the steady state.

We can simplify this process, by plugging in the posterior beliefs to the next period prior beliefs, and get the key equations shown in the paper. These beliefs play an important rule in agents' decision.