

# **Beyond Standards of Proof: Accuracy and Frugality of Different Legal Systems**

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## **Abstract**

Different standards of proof are applied under the common law systems and the civil law systems. The reason is that the common-law adversarial systems rely on litigating parties to produce evidence while the civil-law inquisitorial systems rely on impartial judges to produce evidence, so the collections of evidence presented to the court are largely different. This paper analyzes the adversarial system with its preponderance of the evidence standard and the inquisitorial system with its theoretical optimal standard of proof. Specifically, we model the adversarial litigation as an evidence presentation game in which the two parties produce and present evidence to the court to maximize their expected payoffs. The equilibrium shows that this system may not be the most accurate fact-finding system. In contrast, the inquisitorial fact-finding, which is analyzed analogously as a statistical hypothesis testing problem, may be more accurate. However, the advantage of the adversarial system is that, it deters more frivolous litigations and encourages more settlements, so it is more frugal because of the considerable litigation resources it saves.

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# I. Introduction

Comparative legal studies in recent decades reveal that the requirements of standard of proof are different between the civil law systems and the common law systems. Specifically, there are at least two different standards of proof under the common law systems: the lower standard *preponderance of the evidence* for civil cases and the higher standard of *beyond a reasonable doubt* for criminal cases. Under the civil law systems, however, there is only one *high uniform* standard of proof for both criminal cases and civil cases, such as the *intime conviction*<sup>1</sup> standard in France. Although the difference is salient, relevant studies, especially normative researches, are rare.

Clermont and Sherwin (2002)<sup>2</sup> is one of the pioneering works addressing this difference, and it suggests and discusses several possible explanations. Its final conclusion is that, the high standard of proof in civil law systems enhances the legitimacy of judicial decisions, whereas the same goal is achieved through trials-by-jury in common law systems. This tentative explanation does help us understanding the *high* standard of proof under the civil law systems, but it does not explain why there is a *uniform* standard of proof. Put it another way, the civil law systems can still apply different levels of tough standards without losing legitimacy of its judicial decisions. For example, they can apply beyond a reasonable doubt standard in civil cases and simultaneously apply the even higher standard *beyond any doubt*<sup>3</sup> in criminal cases.

In addition, Clermont and Sherwin (2002) mentions a conventional wisdom justifying different standards of proof for different types of cases. That is, in civil cases, an erroneous decision for the plaintiff (namely, a type-I error) is as harmful as an erroneous decision for the defendant (namely, a type-II error); however, in criminal cases, convicting an innocent is far more costly than acquitting a guilty<sup>4</sup>. Therefore, it is socially desirable to require a higher standard of proof in criminal cases than in civil cases to decrease convicted innocents. Most previous studies on standard of proof are based on this error-cost-ratio approach. However, this approach still cannot explain the *unique* standard of proof under the civil law systems: There is no significant difference in social preference between the civil law societies and common law

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<sup>1</sup> In French, the word “intime” means “inner” and the word “conviction” means “belief”. Therefore, “intime conviction” literally means “inner belief” or “private conviction”. Please refer to the Larousse Online Dictionary at <http://www.larousse.com/en/dictionaries/bilingual>. Some legal dictionaries even directly translate “intime conviction” as “conviction beyond a reasonable doubt”. See Collins Online Dictionary at <http://www.collinsdictionary.com/dictionary/french-english/intime-conviction?showCookiePolicy=true>.

<sup>2</sup> Clermont, K. M. and E. Sherwin (2002), A Comparative View of Standard of Proof, *The American Journal of Comparative Law*, Vol. 50, No. 2, pp. 243-275.

<sup>3</sup> Beyond any doubt was once the standard of proof applied in criminal cases before the current standard beyond a reasonable doubt was developed. For more information about this standard, please see, for example, Morano, A. (1975), A Reexamination of the Development of the Reasonable Doubt Rule, *Boston University Law Review*, Vol. 55, pp. 510-513.

<sup>4</sup> Clermont and Sherwin (2002), pp. 268.

societies<sup>5</sup>. If the error-cost-ratio wisdom held, we should also have seen at least two different standards of proof.

It is therefore worth noting an alternative viewpoint suggested by Judge Posner in his 1999 paper<sup>6</sup>: the common-law adversarial system puts the parties into an arena where they should compete at all costs in searching and presenting evidence to win the case. As a result, the imbalance of parties' litigation resources may significantly bias the fact finder's decision. In criminal cases, such imbalance is much more significant than that in civil cases, because the public prosecutors are endowed with enormous litigation resources from the government which can hardly be matched by the private defendants. Therefore, a high standard of proof, beyond a reasonable doubt, is required to offset such imbalance in criminal cases, whereas a lower standard, preponderance of the evidence, is required in civil cases where the resource imbalance is not significant. On the other hand, the civil-law inquisitorial system requires an impartial judge rather than the litigants to search for evidence, so the imbalance of litigation resources between parties is not a problem. As a result, it is not necessary to require a higher standard of proof in criminal cases than in civil cases.

Following Posner (1999)'s entry point, we are going to show that comparing standards of proof under different legal systems is largely comparing apples and oranges. The adversarial system and the inquisitorial system provide different collections of evidence to the court, so their decision making problems are distinct in essence. Under the adversarial system and the preponderance of the evidence standard, the two litigating parties are actually playing an evidence presentation game in which they produce and present evidence to the court to maximize their expected payoffs from the trial. Under the inquisitorial system, the adjudication decision making is similar to testing a statistical hypothesis based on random sampling, of which the optimal standard of proof can be given by the famous Neyman-Pearson Lemma<sup>7</sup>.

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<sup>5</sup> In criminal cases, people in the civil law world also think that a type-I error is more harmful than a type-II error. For example, a recent empirical study shows that, in Netherland, a typical civil law country, people's subjective error-cost-ratios (the seriousness of a type-I error to the seriousness of a type-II error) mostly fell in the range from 5:1 to 9:1 for different types of crimes. This result is quite close to the theoretical 10:1 "Blackstone Ratio" generally accepted in the common law world. See Keijser et al. (2014), *Wrongful Convictions and the Blackstone Ratio: An Empirical Analysis of Public Attitudes, Punishment & Society*, Vol. 16, No. 1, pp. 32–49. In civil cases, some empirical researches show that the civil law judges also think that a type-I error is as harmful as a type-II error. See, for example, Schweizer, M. (2013), *The Civil Standard of Proof - What Is It, Actually?* MPI Collective Goods Preprint, No. 2013/12. Available at SSRN: <http://ssrn.com/abstract=2311210> or <http://dx.doi.org/10.2139/ssrn.2311210>.

<sup>6</sup> Posner, Richard A. (1999), *An Economic Approach to the Law of Evidence*, *Stanford Law Review*, Vol. 51, No. 6, pp. 1505-1507.

<sup>7</sup> It is worth noting that, under the adversarial system, because parties' evidence production behaviors are affected by the standard of proof, the evidence sample presented to the court is no longer a random sample with respect to standard of proof, so most classic statistics theories including the Neyman-Pearson Lemma are not directly applicable. Therefore, it is not an easy

After examining fact-finding problems under the two legal systems, we move further on to answer a longstanding yet still controversial question. That is, which system, the adversarial or the inquisitorial, is more accurate in finding the truth, and which one is more frugal in resolving disputes. Speaking more generally, which system is superior? Considering the global trend that these two systems are fusing with each other during the last several decades<sup>8</sup>, the answer might be that each system has both advantage and disadvantage. Our theoretical analysis justifies this guess. It shows that, the inquisitorial system is more accurate in finding the truth, whereas the adversarial system is more oriented at resolving disputes. As a byproduct, this study also contributes to the rapid growing literature comparing the adversarial system versus the inquisitorial system.

The discussion of this paper proceeds as follows: Part II reviews relevant literature. Part III analyzes the evidence presentation and adjudication decision making under the adversarial system and the preponderance of the evidence standard. Part IV derives the optimal standard of proof under the inquisitorial system in a statistical way. Part V compares these two systems from the angles of truth-revealing and dispute resolving based on their choices of standard of proof. Part VI provides a summary for this paper.

## II. Literature Review

Based on the topics of this paper, in this section, we are going to review literature in three areas: 1. Theoretical studies on the preponderance of the evidence standard; 2. Normative studies on the determination of the optimal standard of proof; 3. Theoretical studies comparing the adversarial system and the inquisitorial system.

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task to derive the optimal standard of proof under the adversarial system.

<sup>8</sup> The fusion of the adversarial system and the inquisitorial system in fact started very early in history. One opinion even holds that there was neither completely adversarial system nor completely inquisitorial system in history. For example, please see Merryman, J. H. and R. Perez-Perdomo (2007), *The Civil Law Tradition: An Introduction to the Legal Systems of Europe and Latin America*, Stanford University Press, pp. 115-116 (Arguing that “[t]he characterization [of ‘adversarial’ system versus ‘inquisitorial’ system] is quite misleading”, and “the prevailing system in both the civil law and the common law world is the ‘dispositive’ system”).) Whatever the early history was, the fusion between these two systems became much more prominent during the past half century. It is observed that the common law systems imported considerable inquisitorial features to reform its judicial procedures. A most prominent example is the introduction of discovery process. For a more complete list of these reforms, please see Sword, E. E. (1989), *Values, Ideology, and the Evolution of the Adversary System*, *Indiana Law Review*, Vol. 64, pp. 301-355. On the other hand, the civil law systems are becoming more and more adversarial, especially in civil cases. For example, nowadays the French judges are more and more reluctant to use their inquisitorial powers in civil cases worrying “disturb[ing] the balance between the parties”, even if those powers are explicitly guaranteed by the French Code of Civil Procedure. For more details, please see Beardsley, J. (1986), *Proof of Fact in French Civil Procedure*, *The American Journal of Comparative Law*, Vol. 34, No. 3, pp. 459-486.

## II. 1. The Preponderance of the Evidence Standard

Compared to the criminal standard beyond a reasonable doubt, historical studies on the civil standard preponderance of the evidence are quite limited. Now we only know that this standard was firstly enunciated in the nineteenth century<sup>9</sup>, possibly because of the increasing needs to instruct civil juries<sup>10</sup>.

Similarly, most economics studies on standard of proof focus on criminal standard rather than civil standard. The most conventional theory justifying the preponderance of the evidence standard is the previously mentioned error-cost-ratio theory. However, there are two preliminary conditions for this theory to hold: first, the evidence available to the court does not vary with the required standard of proof; second, the lawsuits going to trial does not vary with the required standard of proof. However, both of the preliminary conditions have some logic flaws. For example, if the plaintiff is required to prove the defendant's negligence beyond a reasonable doubt rather than to be "more likely true than not"<sup>11</sup>, some plaintiffs may choose to present more evidence to the court if they are really eager to win the case, while some other plaintiffs may choose to repeal the complaint because it is very unlikely for them to win.

Besides, there are some theoretical works studying the preponderance of the evidence standard together with other legal rules, such as Demougin and Fluet (2005, 2006)<sup>12</sup>. Demougin and Fluet (2005) shows that the strict liability rule does not always induce higher level of deterrence than the negligence rule. When negligence rule provides higher level of deterrence and if judicial errors do not matter, the preponderance of the evidence is optimal. However, if judicial errors do matter, it is better to apply a higher standard of proof with an appropriate assignment of the persuasion burden. Demougin and Fluet (2006) shows that, the preponderance of the evidence standard and the common law exclusionary rules jointly provide the maximum level of deterrence no matter which party bears the burden of persuasion. Under other levels of standard of proof, however, the deterrence level is largely affected by the allocation of the persuasion burden.

## II. 2. Normative Studies on Standard of Proof

Based on our discussions in the previous section, we learn that the preponderance of the evidence standard is optimal under some circumstances but not optimal under other circumstances. That is to say, whether a

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<sup>9</sup> See Note 87 of Berman, H. J. and C. J. Reid (1996), *The Transformation of English Legal Science: From Hale to Blackstone*, *Emory Law Journal*, Vol. 45, No. 2, on Page 482.

<sup>10</sup> See Clermont, K. M. (2004), *Standard of Proof in Japan and the United States*, *Cornell International Law Journal*, Vol. 37, on Page 274.

<sup>11</sup> This is a common explanation of the preponderance of the evidence standard in jury instructions in the United States.

<sup>12</sup> Demougin, D. and C. Fluet, (2005) *Deterrence versus Judicial Error: A Comparative View of Standards of Proof*, *Journal of Institutional and Theoretical Economics (JITE)*, 161(2): 193-206; Demougin, D. and C. Fluet, (2006) *Preponderance of evidence*, *European Economic Review*, 50: 963-976.

standard of proof is optimal is largely decided by the specific problem it is going to solve. In economics words, whether a standard of proof is optimal is largely decided by the legislator's utility function, or more specifically the social welfare function.

The previously stated error-cost-ratio theory is in fact based on a social welfare function in which only adjudication errors matter. However, as we previously pointed out, if we add other welfare concerns into the social welfare function (such as the costs of producing evidence or the deterrence of undesired behaviors), the optimal standard of proof may be different. In the follows, we will review some representative normative studies based on other forms of social welfare functions.

The most representative normative studies on standard of proof is given by Kaplow (2011a, 2011b)<sup>13</sup>. In this series of papers, the welfare function incorporates people's behaviors and legal enforcement costs; however, adjudication errors are not considered. His analyses show that, when the standard of proof is low, more undesired behaviors will be deterred. However, at the same time, some desired behaviors may be "chilled", because the enforcer and fact-finder might confuse them with undesired behaviors and thus punished them<sup>14</sup>. Therefore, the optimal standard of proof is determined by the tradeoff between "detering undesired behaviors" and "chilling desired behaviors".

## II. 3. Comparing the Adversarial and Inquisitorial Systems

Milgrom and Roberts (1986)<sup>15</sup> is one of the earliest works studying information revealing systems. This paper implies that, to reach a correct decision, the fact-finder needs to be more sophisticated under the inquisitorial system than under the adversarial system. In addition, when information revealing is limited, a sophisticated fact-finder can always reach more correct decisions under the adversarial system than under the inquisitorial system.

Shin (1998) argues that the adversarial system is superior over the inquisitorial system because the fact finder can get more information when there is no evidence provided to the court. For example, when the fact finder knows that the defendant has better access to the truth but receives no evidence from the defendant, it is highly possible that the defendant is hiding the truth which is unfavorable to her.

Dewatripont and Tirole (1999)<sup>16</sup> argues that, when information cannot be manipulated, it is easier and

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<sup>13</sup> Kaplow, L., (2011a), On the Optimal Standard of Proof, *Journal of Political Economy*, Vol. 119, No. 6, pp. 1104-1140; Kaplow, L., (2011b), Optimal Proof Standards, Deterrence, and the Chilling of Desirable Behavior, *American Economics Review*, Vol. 101, No.3, pp. 277-280; Kaplow, L., (2012), Standard of Proof, *The Yale Law Journal*, Vol. 121, pp. 738-859.

<sup>14</sup> Kaplow (2011a) gives some examples of these costs, such as that "health care providers might avoid treatment of high-risk patients for fear of erroneous malpractice liability".

<sup>15</sup> Milgrom, P. and J. Roberts, Relying on the Information of Interested Parties, *The RAND Journal of Economics*, Vol. 17, No. 1 (Spring, 1986), pp. 18-32.

<sup>16</sup> Dewatripont, M. and J. Tirole, Advocates, *Journal of Political Economy*, Vol. 107, No. 1 (February 1999), pp. 1-39.

cheaper to design an incentive-compatible payoff regime under the adversarial system than under the inquisitorial system, because the inquisitorial system requires the investigation agent to pursue two sharply contradicting positions at the same time. As a result, the adversarial system is superior because of more revealed information and lower costs. However, if information can be manipulated (mainly in the form of concealing), the inquisitorial system may be superior when an erroneous passive decision<sup>17</sup> is considerably more harmful than an erroneous active decision<sup>18</sup>.

Nakao and Tsumagari (2012)<sup>19</sup> follows and extends Dewatripont and Tirole (1999)'s framework. This paper shows that when evidence cannot be manipulated, the inquisitorial system is inferior not only because it requires the investigation agent to pursue two types of sharply contradicting evidence at the same time (as suggested by Dewatripont and Tirole (1999)), but also because it requires the judge to evaluate her own performance. Such "Trilemma" does not exist in the adversarial system, so it is much easier to induce higher level of investigation efforts. However, just as suggested in Dewatripont and Tirole (1999), when parties can manipulate evidence, the adversarial system may be inferior because it may waste resources without improving the accuracy of the adjudication.

To sum up, most previous comparison studies agree that, when evidence is impossible to manipulate, the adversarial system is superior over the inquisitorial system; however, such conclusion may not stand if litigants can manipulate evidence.

## III. The Adversarial Fact-finding in Civil Cases

In this section, we will analyze evidence presentation and adjudication decision making under the adversarial system and the preponderance of the evidence standard. Before we move on to the formal game theory analysis, it is important to define some concepts and make some clarifications.

### III. 1. Concepts and Clarifications

In this section, we will mainly discuss the following ideas: 1. The fact-finding process answers a binary question; 2. The evidence, which is the only independent variable in the "standard of proof" function, is related to the truth<sup>20</sup>; 3. The standard of proof, which is in essence our way of explaining evidence, can be thought as a function.

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<sup>17</sup> In Dewatripont and Tirole (1999), this is termed as "inertia".

<sup>18</sup> In Dewatripont and Tirole (1999), this is termed as "extremism".

<sup>19</sup> Nakao, K. and M. Tsumagari, The Inquisitor Judge's Trilemma, *Review of Law and Economics*, Vol. 8, No. 1 (2012), pp. 137-159.

<sup>20</sup> In judicial practices, the terms "fact" and "truth" are usually interchangeable. Here, to avoid confusion, we refer "truth" as the underlying objective nature which the fact-finding process pursues, and we name "fact" as our subjective conclusion of the fact-finding process.

### III. 1. A. The fact-finding process answers a binary question

In judicial practices, there are countless specific questions of fact. They can be generally categorized into two types: one type concerns the existence of an event. For example, whether the plaintiff's loss was caused by the defendant's negligence. The answer to this type of questions is binary "yes" or "no". The other type concerns the degree of an event. For example, how much was the plaintiff's loss incurred by the defendant's negligence. The answer of this type of questions is mostly a cardinal number.

Previously, most law and economics models on fact-finding study the second type of questions. For example, in Froeb and Kobayashi (1996)<sup>21</sup> and Froeb and Kobayashi (2001)<sup>22</sup>, the court estimates the plaintiff's deserved share within the disputed value (and the rest is deserved by the defendant), so the court is essentially *dividing* the disputed value between the plaintiff and the defendant. However, in judicial practices, much more questions belong to the first type rather than the second type, and even if a second-type question is raised, it is usually based on the solution of an antecedent first-type question. For example, the court will not move on to deliberate the amount of compensation in a tort lawsuit if the fact finder does not agree with the existence of the tort at all.

In this paper we will focus on the first type of questions of fact. To make our discussion more straightforward, we simplify the question of fact to be "whether the defendant is truly liable". However, such literal fact-finding problem cannot get us very far in understanding fact-finding. Instead, we may wonder if we can express this literal question in a mathematical way which provides more possibility for further analysis. In fact, there is a widely adopted modeling method to mathematize the fact-finding problems: the evidence or case strength (a generalized term which can refer to the quantity, the quality, or both quantity and quality of the evidence) is a random variable, and this random variable follows different distributions when the defendant is truly liable or truly not liable. For a most representative example of this thinking mode, please see Figure 1 of Kaplow (2012)<sup>23</sup>, in which the distributions of the "evidence variable" for truly liable defendants and truly not liable defendants are illustrated by two normal-like distributions with different means and (probably) the same variance. Therefore, with proper assumptions and reasoning, we would like to transform the question "whether the defendant is truly liable" into the

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<sup>21</sup> Froeb, L. and B. Kobayashi (1996), Naive, Biased, Yet Bayesian: Can Juries Interpret Selectively Produced Evidence? *Journal of Law, Economics, & Organization*, Vol. 12, No. 1, pp. 257-276. This paper focuses mainly on the accuracy of fact finder' decision under the adversarial system. It shows that, when the fact finder is both naïve (that is to say, the fact finder "naively" takes the evidence in the court as an random sample from the underlining statistical population) and Bayesian (that is to say, the fact finder utilizes the Bayesian's method to make an inference), she can reach an accurate estimation of the truth.

<sup>22</sup> Froeb, L. and B. Kobayashi (2001), Evidence Production in Adversarial vs. Inquisitorial Regimes, *Economics Letters*, Vol. 70, pp. 267–272. In this paper, the two different systems along with their decision rules are compared; however, the decision rules are exogenously given with no explanation. This paper compares the means and variances of the estimations of the truth under the two regimes and concludes that neither regime dominates the other one.

<sup>23</sup> Kaplow, L. (2012), Standard of Proof, *The Yale Law Journal*, Vol. 121, Page 757.

following mathematical question “which distribution (or group of distributions) does the evidence variable follow”. (Such assumption is provided by Assumption 3 in Section III. 2)

### **III. 1. B. Evidence is related to the truth**

Fact finding conclusions should be based on evidence, but not on “trial by ordeal” or “tossing a coin”. This is simply because the evidence more or less reveals the truth. In other words, if some information cannot improve our cognition of the truth, it should not be admitted as evidence, such as the hearsay evidence. Therefore, it is necessary for us to discuss this inner relationship between the truth and evidence in our model.

Previously, we have already discussed our intention to express the truth in a mathematical way. Most preferably, we would like to model the truth and its alternatives as different distributions of some “evidence variable”. In order to do this, we firstly need to transform the qualitative evidence in judicial practices into quantitative variables. Furthermore, in mathematics, the observations of a random variable are generated according to the true distribution. Analogously, we can think in the following way: the evidences are the observations according to the defendant’s “true type” (i.e. “truly not liable” or “truly liable”). Put it another way, the defendant’s “true type” will affect our chances of observing different types of evidence, and therefore we can model the inner relationship between the truth and evidence.

Then how does the defendant’s “true type” affect our chances of observing different types of evidence? To answer this question, we need to reason one more step backward and make some assumptions about how we observe evidence, i.e., how the investigation process works in our model (Assumption 1 and 2 in Section III. 2).

### **III. 1. C. Standard of proof is a function**

In essence, the fact-finding process is explaining the evidence presented to the court. If we think evidence as the independent variable and our conclusion as the dependent variable, our way of explaining evidence is a (surjective) function connecting the independent variable and the dependent variable, because on the court any collection of evidence arrives in one and only one conclusion, either for the plaintiff or for the defendant.

In judicial practices, it is exactly the standard of proof that connects evidence and our conclusion, which implies that the standard of proof can be modeled as a function (most likely as a “decision function” which we will discuss in details later). Furthermore, different standards of proof are different functions, because the same collection of evidence may result in different conclusions under different standards of proof (for example, the different outcomes of the O. J. Simpson Trials).

To sum up, the fact-finding process pursues the binary truth “whether the defendant is truly liable or truly not liable”. The “true type” (which is going to be defined by Assumption 3 in Section III. 2) of the

defendant affects what evidence we can get from investigation (which is going to be defined by Assumption 1 and 2 in Section III. 2). The adjudication decision function, or the standard of proof, maps the presented evidence to the adjudication conclusion. Sometimes the adjudication conclusion is consistent with the underlying truth, but sometimes it is not. When the adjudication conclusion is not consistent with the underlying truth, an adjudication error occurs: if a truly not liable defendant is found liable, a type-I error occurs; if a truly liable defendant is found not liable, a type-II error occurs.

## III. 2. Assumptions

**Assumption 1 (The Investigation Assumption):** (a) Every investigation produces exactly one piece of evidence which indicates either liability or no liability of the defendant. (b) The results of different investigations are independent from each other. (c) Every piece of evidence indicating liability of the defendant is equally powerful, and every piece of evidence indicating no liability of the defendant is also equally powerful.

In modern trials, a question of fact is usually decomposed into many different and independent elements. For example, to establish a defective product liability claim, the following elements must be proved: 1. The plaintiff was injured or suffered losses; 2. The product is defective; 3. The defect caused the plaintiff's injury; 4. The plaintiff was using the product as it was intended. To prove each of these elements, there are many conventional types of evidence, so practical investigations are usually targeting at one specific type of evidence. For example, to prove or disprove the "defectiveness" element, a common practice for the litigants is to consult an expert in product manufacture. However, this expert may either find some manufacturing defects or not, and his conclusion or opinion can be treated as ONE piece of evidence (expert witness) if presented to the court. In sum, the expert witness may indicate either liability or no liability of the defendant on the manufacturing defect aspect. This justifies our Assumption 1(a). Assumption 1(b) is also easy to justify: the evidence regarding different elements are largely independent with each other. If not, it would not be necessary to separate those elements. For example, a piece of evidence indicating "defectiveness" is very unlikely to influence the result of an investigation regarding whether the plaintiff was injured, and vice versa. Finally, since those elements are all necessary in proving the claim, we cannot say that a piece of evidence proving Element A is more powerful than a piece of evidence proving Element B, not to mention that different evidences usually support the same element, which justifies our Assumption 1(c).

After describing the investigations in our model, we take a big step to simplify our analysis, which is given by Assumption 2:

**Assumption 2 (The Identical Distribution Assumption):** For any given case, every investigation generates either type of evidence indicating the defendant's liability or no liability according to an identical distribution.

Putting Assumption 1 and Assumption 2 together, we have the following preliminary results: Based on

Assumption 1 (a), we know that the result of one investigation can be modeled with Bernoulli distribution: with a probability of  $\theta$  ( $0 \leq \theta \leq 1$ ), the investigation results in a piece of evidence indicating liability of the defendant, while with a probability of  $(1 - \theta)$ , the investigation results in a piece of evidence indicating no liability of the defendant. Based on Assumption 2, those Bernoulli distributions regarding the same case are all identical, i.e., the values of  $\theta$ s are the same. Based on Assumption 1(b), the Bernoulli distributions regarding the same case are independent with each other. As a result, for the same case, the result of every investigations follows an independent and identical Bernoulli distribution. Suppose for a given case a total number of  $N$  investigations are conducted, which result in  $n_p$  pieces of evidence indicating liability (or for the plaintiff) and  $n_d$  pieces of evidence indicating no liability (or for the defendant). According to the relationship between Bernoulli distribution and binominal distribution, we know  $n_p \sim \mathcal{B}(N, \theta)$  ( $\mathcal{B}(N, \theta)$  stands for the binominal distribution with parameters  $N$  and  $\theta$ . If  $N = 1$ , it is a Bernoulli distribution),  $n_d \sim \mathcal{B}(N, 1 - \theta)$  and  $n_p + n_d = N$ .

Note that the value of parameter  $\theta$  is identical for every investigation regarding the same case, but it is not necessarily identical for investigations regarding different cases. Namely, the parameter  $\theta$  characterizes a case. The value of  $\theta$  actually indicates how easy the plaintiff can turn his investigation efforts into evidence favoring him. With the same investigation effort, the larger  $\theta$  is, the more evidence indicating the defendant's liability can be obtained by the plaintiff. Briefly speaking,  $\theta$  indicates how strong the plaintiff's claim is: the larger  $\theta$  is, the easier the plaintiff can prove his claim, and the harder the defendant can attack the plaintiff's claim. Accordingly, we name the parameter  $\theta$  as the **case-strength parameter**.

Note further that the case-strength parameter  $\theta$  describes our chances of observing different types of evidence during investigation. Following our argument in the previous section, the "true type" of the defendant determines our chances of observing different types of evidence during investigation, so the "true type" of the defendant determines the value of  $\theta$ . In turn, the value of  $\theta$  indicates the "true type" of the defendant. In the following assumption, we define the relationship between the "true type" of the defendant and the value of  $\theta$ .

**Assumption 3 (The Case-strength Parameter Assumption):** Any investigation targeting at a truly liable defendant is more likely to result in a piece of evidence indicating liability of the defendant than to result in a piece of evidence indicating no liability of the defendant. That is to say, if the defendant is truly liable, the case-strength parameter is greater than 0.5.<sup>24</sup> Any investigation targeting at a truly not liable defendant is more likely to result in a piece of evidence indicating no liability of the defendant than to result in a piece of evidence indicating liability of the defendant. That is to say, if the defendant is truly not liable, the case-strength parameter is less than 0.5. For completeness of mathematics, we assume that

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<sup>24</sup> This is easy to see: for an investigation, the probability of finding a piece of evidence indicating liability is  $\theta$ , and the probability of finding a piece of evidence indicating no liability is  $1 - \theta$ . According to this assumption, if the defendant is truly liable, we have  $\theta > 1 - \theta$ , so  $\theta > 0.5$ .

the defendant is also truly not liable when the case-strength parameter equals to 0.5.

Mathematically:

$$\theta > 0.5 \Leftrightarrow \text{the defendant is truly liable}$$

$$\theta \leq 0.5 \Leftrightarrow \text{the defendant is truly not liable}$$

In addition, we assume that both litigating parties know the value of  $\theta$  and thus the true type of the defendant, but the fact finder does not know it (because she does not know the defendant's true type). That is exactly the reason to conduct fact-finding.

The justification of this assumption is mainly based on the following common sense. From a methodological point of view, an investigation method, if properly designed and verified through years of practices, should be more likely to lead us to the truth rather than to deviate us from it. If not, such investigation method should be abandoned by the legal system. To see this point, let us conduct a thought experiment. Suppose the investigation resources in society is so limited that only one investigation can be afforded for each case, and the fact finder decides the case according to the result of the one-shot investigation (i.e., the plaintiff wins the case if the investigation results in a piece of evidence indicating liability, and vice versa). Then a "meaningful" investigation should result in a piece of evidence indicating liability with a probability higher than 0.5 if the defendant is truly liable, and result in a piece of evidence indicating no liability with a probability lower than or equal to 0.5 if the defendant is truly not liable. If not, the investigation is less accurate than "tossing a fair coin" to decide the case, by which the deserving party has an exact 50% chance to win the case. As a result, this investigation method should be abandoned because it wastes social resources but makes the truth more obscure.

Based on Assumption 3, we can finally transform the literal fact-finding problem "whether the defendant is truly liable or not" into a mathematical problem examining "whether the case-strength parameter  $\theta$  is greater than 0.5".

**Assumption 4 (The Preponderance of the Evidence Standard Assumption):** After the parties present evidences to the court, the fact finder makes the decision according to the following rule: If  $n_p > n_d$ , the plaintiff wins the case; if  $n_p \leq n_d$ , the defendant wins the case. That is to say, the party who has the preponderance of the evidence wins the case<sup>25</sup>. Mathematically, the standard of preponderance of the evidence takes the form of a decision function as follows:

$$\Lambda = \begin{cases} 1 & n_p > n_d \\ 0 & n_p \leq n_d \end{cases}$$

In which  $\Lambda = 1$  means that the court renders a decision for the plaintiff, whereas  $\Lambda = 0$  means that the

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<sup>25</sup> Although no party has the preponderance of the evidence if  $n_p = n_d$ , it is the convention in the common law world to render a decision for the defendant in this circumstance. See, for example, (TBA).

court renders a decision for the defendant.

Although there was not, is not and perhaps will never be a final consensus on the true meaning of the phrase “preponderance of the evidence”, sticking to its literal meaning is always the least risky way to understand and apply it. According to the Merriam-Webster Online Dictionary, preponderance means “a greater amount or number of something”<sup>26</sup>. Putting this interpretation and our previous assumptions together, it is not difficult to derive the above stated form of the preponderance of the evidence standard.

**Assumption 5 (Parties’ Strategies Assumption):** Assume that the standard of proof and the case-strength parameter  $\theta$  are common knowledge of both parties before they make any decision. The plaintiff moves first to decide whether to file the lawsuit. Once a case is filed, the defendant chooses whether to settle with the defendant out of court. If the case is not settled, it will go to trial, and parties start preparing evidence to be presented to the court. Notably, we assume that the parties decide how many pieces of evidence to *present* to the court (but not how many investigations to conduct) and estimate the total evidence production cost *ex ante*<sup>27</sup>. Let’s denote the number of evidence presented by the plaintiff as  $n_p$ , and the number of evidence presented by the defendant as  $n_d$ <sup>28</sup>. Let’s further assume that every investigation is equally costly no matter who conducts it, and the constant evidence production cost is assumed to be 1. Both parties know nothing about the other party’s investigation progress until the trial (i.e., there is no discovery process)<sup>29</sup>. On the trial, both parties present evidence to the court, and the court makes the decision according to the preponderance of the evidence standard.

According to Assumption 5, in the evidence presentation game, given the plaintiff’s strategy  $n_p$ , the total number of investigations needed to be conducted by the plaintiff (denoted as  $N_p$ ) equals to  $n_p$  plus a

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<sup>26</sup> <http://www.merriam-webster.com/dictionary/preponderance>, assessed on

<sup>27</sup> This assumption means that the parties keep on producing evidence until they obtain the number of evidence they want. In Froeb and Kobayashi (1996, 2001) and other game theory studies, this evidence production behavior is named as “stopping game”.

<sup>28</sup> According to the standard of proof previously defined in Assumption 4, it is not hard to see that all  $n_p$  pieces of evidence from the plaintiff indicate liability of the defendant, and all  $n_d$  pieces of evidence from the plaintiff indicate no liability of the defendant. This is also in accordance with the common sense that in the adversarial system litigating parties will only present evidences favoring themselves.

<sup>29</sup> There are reasons mainly in three different aspects to make this assumption. First, as many observers pointed out, the discovery process is not an indigenous component of the adversarial systems. Rather, it was imported from the civil law systems as a way to improve the common law systems. Second, in judicial practices, the most important function of the discovery process is to reveal the true strength of the plaintiff’s case (or the value of  $\theta$  in our assumptions) to both parties. In other words, the discovery process is unnecessary in our model setting once we assume that the value of  $\theta$  is common knowledge to both parties at the very beginning. Third, from an efficiency point of view, introducing the discovery process may not be an improvement of the legal system according to our model setting. This is because introducing discovery process into this model turns the model into a “dollar auction” problem which, as shown in many behavioral economics studies, usually results in an inefficient outcome. See, for example, (TBA).

negative binominal random variable with a distribution  $\mathcal{NB}(n_p, \theta)$  and an expected value  $n_p \cdot \frac{1-\theta}{\theta}$ . As a result, if the plaintiff decides to present  $n_p$  pieces of evidence to the court, her expected evidence production costs is  $N_p = n_p + n_p \cdot \frac{1-\theta}{\theta} = \frac{n_p}{\theta}$ . Similarly, if the defendant decides to present  $n_d$  pieces of evidence to the court, her expected evidence production costs is  $\frac{n_d}{1-\theta}$ .

**Assumption 6 (Parties’ Incentive Assumption):** If the plaintiff does not file the lawsuit, both parties get a payoff of 0. If the plaintiff files the case, the defendant can choose to settle with the plaintiff out of court by paying the plaintiff the disputed value  $B$  (stands for “Benefit”). If a lawsuit is settled, no litigation cost will be incurred. If the case goes to trial and the plaintiff prevails, the defendant is required to pay  $B$  to the plaintiff; if the defendant prevails, she needs to pay nothing. All parties bear their own investigation costs no matter what the outcome of the trial is (i.e., there is no English rule of “loser pays”).

Mathematically, given  $n_p$ , the plaintiff’s expected payoff from the trial is:

$$u_p = \begin{cases} 0, & \text{no filing} \\ B, & \text{settlement} \\ B - \frac{n_p}{\theta}, & \text{wins the trial} \\ -\frac{n_p}{\theta}, & \text{loses the trial} \end{cases}$$

And given  $n_d$ , the defendant’s expected payoff from the trial is:

$$u_d = \begin{cases} 0, & \text{no filing} \\ -B, & \text{settlement} \\ -\frac{n_d}{1-\theta}, & \text{wins the trial} \\ -B - \frac{n_d}{1-\theta}, & \text{loses the trial} \end{cases}$$

### III. 3. Analyzing the Adversarial System with the Standard of Preponderance of the Evidence

To analyze the parties’ evidence presentation behavior under the adversarial system, firstly we need to analyze their decisions of filing or settling the case. However, if we want to know their decisions of filing or settling the case, we need to know their expected payoffs from the trial (that is to say, we need to “looking forward and reasoning backward”), which is determined by the equilibrium of the evidence presentation game. The equilibrium is given by the following proposition.

(To simplify calculation, we will treat the values of  $n_p$  and  $n_d$  as continuous variables rather than

discrete variables, just as Froeb and Kobayashi (1996) does.)

**Proposition 1:** The only equilibrium for the evidence presentation game is a mixed strategy equilibrium described as follows:

When  $\theta \leq 0.5$ , the plaintiff randomizes  $n_p$  according to the following cdf:

$$F_p(x) = \frac{(1 - 2\theta)B + x}{(1 - \theta)B}, x \in [0, \theta B]$$

And the defendant randomizes  $n_d$  according to the following cdf:

$$F_d(x) = \frac{x}{\theta B}, x \in [0, \theta B]$$

When  $\theta > 0.5$ , the plaintiff randomizes  $n_p$  according to the following cdf:

$$F_p(x) = \frac{x}{(1 - \theta)B}, x \in [0, (1 - \theta)B]$$

And the defendant randomizes  $n_d$  according to the following cdf:

$$F_d(x) = \frac{(2\theta - 1)B + x}{\theta B}, x \in [0, (1 - \theta)B]$$

The proof of Proposition 1 is provided in the Appendix Section.

The mixed strategy equilibrium can be explained as follows: because there is no discovery process, each party knows nothing about her opponent's presentation decision before the trial. If one party's presentation decision is certain, it will be a great disadvantage to herself as well as a great advantage to her opponent, because her opponent can either win the case with the minimum cost or cause a huge waste to her (as we show in the appendix). Therefore, it is to every party's very benefit to make her own decision uncertain.

However, parties' decisions can still be "predicted" even though they are uncertain. In fact, both parties will randomize their decisions according to some equilibrium distributions. With these randomizations, both parties' payoffs are constant numbers irrelevant to their presentation decisions. As a result, both of them have no incentive to deviate from this equilibrium.<sup>30</sup>

With Proposition 1, we can derive the following useful corollaries:

**Corollary 1-1:** The plaintiff's unconditional probability to win the case (denoted as  $\mathcal{P}_p$ ) is:

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<sup>30</sup> For a more detailed discussion of the philosophy behind mixed strategy equilibrium, please see Bicchieri, C. (1998), Decision and Game Theory, Routledge Encyclopedia of Philosophy: Index, pp. 826-827.

$$\mathcal{P}_p = \begin{cases} \frac{1}{2} \cdot \frac{\theta}{1-\theta}, \theta \leq 0.5 \\ 1 - \frac{1}{2} \cdot \frac{1-\theta}{\theta}, \theta > 0.5 \end{cases}$$

And the defendant's unconditional probability to win the case is:

$$\mathcal{P}_d = \begin{cases} 1 - \frac{1}{2} \cdot \frac{\theta}{1-\theta}, \theta \leq 0.5 \\ \frac{1}{2} \cdot \frac{1-\theta}{\theta}, \theta > 0.5 \end{cases}$$

Proof: From Proposition 1, it is easy to see that the probability for the plaintiff to win the case conditioning on that the defendant presents  $n_d$  pieces of evidence is:

$$1 - F_p(n_d) = \begin{cases} \frac{\theta B - n_d}{(1-\theta)B}, n_d \in [0, \theta B] \text{ and } \theta \leq 0.5 \\ \frac{(1-\theta)B - n_d}{(1-\theta)B}, n_d \in [0, (1-\theta)B] \text{ and } \theta > 0.5 \end{cases}$$

Therefore, the unconditional probability for the plaintiff to win the case is:

$$\begin{cases} \int_0^{\theta B} \frac{\theta B - n_d}{(1-\theta)B} dF_d(n_d), \theta \leq 0.5 \\ \int_0^{(1-\theta)B} \frac{(1-\theta)B - n_d}{(1-\theta)B} dF_d(n_d), \theta > 0.5 \end{cases}$$

Substitute the defendant's strategy  $F_d$  into the integrations above, it is straightforward to get the unconditional probability for the plaintiff to win the case is:

$$\mathcal{P}_p = \begin{cases} \frac{1}{2} \cdot \frac{\theta}{1-\theta}, \theta \leq 0.5 \\ 1 - \frac{1}{2} \cdot \frac{1-\theta}{\theta}, \theta > 0.5 \end{cases}$$

On the other hand, the defendant's unconditional probability to win the case is just 1 minus the unconditional probability for the plaintiff to win the case. Corollary 1-1 is thus proved.

**Corollary 1-2:** Under the adversarial system and the standard of preponderance of the evidence, the probabilities of adjudication errors are:

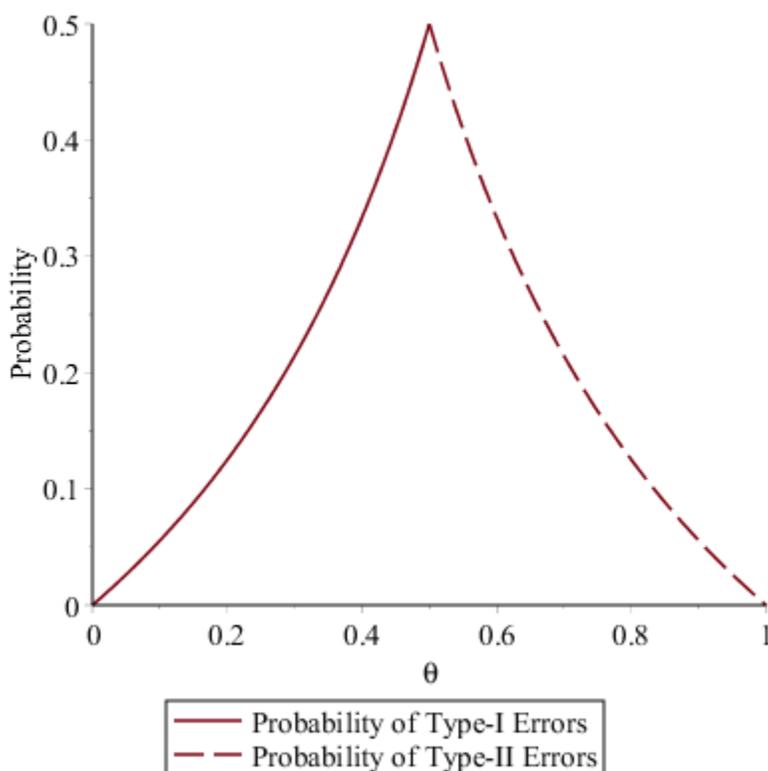
$$\text{Type-I error: } \frac{1}{2} \cdot \frac{\theta}{1-\theta}, \theta \leq 0.5$$

$$\text{Type-II error: } \frac{1}{2} \cdot \frac{1-\theta}{\theta}, \theta > 0.5$$

Proof: A type-I error means that the defendant loses the case when she is truly not liable (which is equivalent to  $\theta \leq 0.5$  according to Assumption 3), and a type-II error means that the defendant wins the case when she is truly liable (which is equivalent to  $\theta > 0.5$  according to Assumption 3). Therefore, from Corollary 1-1, it is straightforward to get the probabilities for the two types of adjudication errors.

Putting type-I errors and type-II errors together, the total probability of adjudication errors under the adversarial system and the preponderance of the evidence standard varies with the value of  $\theta$ , as shown in Figure 1:

Figure 1 Probability of Adjudication Errors under the Adversarial System



**Corollary 1-3:** The plaintiff's unconditional expected litigation cost (denoted as  $C_p$ ) is:

$$C_p = \begin{cases} \frac{1}{2} \cdot \frac{\theta}{1-\theta} B, \theta \leq 0.5 \\ \frac{1}{2} \cdot \frac{1-\theta}{\theta} B, \theta > 0.5 \end{cases}$$

The defendant's unconditional expected litigation cost is:

$$C_d = \begin{cases} \frac{1}{2} \cdot \frac{\theta}{1-\theta} B, \theta \leq 0.5 \\ \frac{1}{2} \cdot \frac{1-\theta}{\theta} B, \theta > 0.5 \end{cases}$$

Proof: When the plaintiff decides to present  $n_p$  pieces of evidence, her expected litigation cost is  $\frac{n_p}{\theta}$ . Therefore, her unconditional litigation cost is:

$$\begin{cases} \int_0^{\theta B} \frac{n_p}{\theta} dF_p(n_p), \theta \leq 0.5 \\ \int_0^{(1-\theta)B} \frac{n_p}{\theta} dF_p(n_p), \theta > 0.5 \end{cases}$$

Substitute the plaintiff's strategy  $F_p$  into the integrations above, we have:

$$C_p = \begin{cases} \frac{1}{2} \cdot \frac{\theta}{1-\theta} B, \theta \leq 0.5 \\ \frac{1}{2} \cdot \frac{1-\theta}{\theta} B, \theta > 0.5 \end{cases}$$

Similarly, we can derive  $C_d$ .

Corollary 1-1, 1-2 and 1-3 together give us a very important insight about the adversarial system and the preponderance of the evidence standard. That is, the probabilities of adjudication errors are only based on the case strength parameter  $\theta$  but not based on the disputed value  $B$ . On the other hand, however, the litigation costs of society is a strictly increasing function with respect to the disputed value. As a result, the accuracy of the adversarial fact-finding does not improve with respect to society's investigation efforts, which is a very astonishing result. In Section V, we will look into this phenomenon further by comparing the accuracy of the adversarial system and the inquisitorial system.

**Corollary 1-4:** The plaintiff's unconditional expected payoff from litigation is:

$$\Pi_p = \begin{cases} 0, & \theta \leq 0.5 \\ \frac{2\theta - 1}{\theta} B, & \theta > 0.5 \end{cases}$$

And the defendant's unconditional expected payoff from litigation is:

$$\Pi_d = \begin{cases} -\frac{\theta}{1-\theta} B, & \theta \leq 0.5 \\ -B, & \theta > 0.5 \end{cases}$$

Proof: We know that  $\Pi_p = \mathcal{P}_p \cdot B - C_p$ . Substitute  $\mathcal{P}_p$  and  $C_p$  which we derived in Corollary 1-1 and

1-3, and it is straightforward to get:

$$\Pi_p = \begin{cases} 0, & \theta \leq 0.5 \\ \frac{2\theta - 1}{\theta} B, & \theta > 0.5 \end{cases}$$

Similarly, we can derive the value of  $\Pi_d$ .

**Corollary 1-5:** Under the adversarial system and the preponderance of the evidence standard, when the defendant is truly not liable, the plaintiff is indifferent between filing the lawsuit or not; when the defendant is truly liable, the defendant is indifferent between settling out of court or not.

Proof: From Corollary 1-4, we know that when the defendant is truly not liable, the plaintiff gets an expected payoff of zero from litigation, which is equal to her payoff from not filing the lawsuit. Therefore she is indifferent between filing the case or not. On the other hand, when the defendant is truly liable, her expected payoff from trial is equal to her settlement payoff ( $-B$ ), so she is indifferent between settling out of the court or not.

The implication behind Corollary 1-4 and 1-5 is also very insightful: under the adversarial system and the preponderance of the evidence standard, the party who is not backed by the truth cannot get better off through litigation. As a result, the adversarial system on one hand discourages frivolous litigations and on the other hand encourages settlements. In other words, the adversarial system is more oriented at dispute-resolving.

## IV. The Optimal Standard of proof under the Inquisitorial System

In the following paragraphs, we will discuss the fact-finding problem under the inquisitorial system. From the discussion associated with Assumption 3 in Section III. 2, we know that the literal fact-finding problem “whether the defendant is truly liable or not” is equivalent to a mathematical problem examining “whether the case-strength parameter  $\theta$  is greater than 0.5”. Under the inquisitorial system, because the impartial judge conducts investigation by herself, the investigation process is equivalent to a random sampling<sup>31</sup>. As a result, it is possible to analyze the inquisitorial fact-finding analogously as a statistical hypothesis testing problem, but it is not possible for the adversarial system because the evidence sample is not random with respect to the standard of proof. Specifically, under the inquisitorial system, the judge in fact solves the following hypothesis testing problem:

$$\mathbf{H}_0: \theta \leq 0.5 \text{ versus } \mathbf{H}_1: \theta > 0.5$$

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<sup>31</sup> See Page 1482 of Posner (1999): “If the searcher cannot determine in advance which evidence is most likely to be fruitful, his search procedure will resemble random sampling”.

In the analyses of the inquisitorial fact-finding, we still use Assumption 1, 2 and 3 from Section III. 2, because those assumptions are all about investigation “technology” which is not different between the adversarial system and the inquisitorial system. However, these two systems do have some differences as we discussed earlier, and to reflect these differences we substitute the rest assumptions in Section III. 2 with the following new assumptions.

## IV. 1. New Assumptions

**Assumption 4’ (The Standard of Proof Assumption):** The standard of proof, as a decision function, requires that the conclusion of fact-finding should only be based on the evidences presented to the court. Because evidences of the same type are equally powerful (from Assumption 1(c)), the independent variables in the decision function only include the number of evidences indicating liability (denoted as  $n_p$ ) and the number of evidence indicating no liability (denoted as  $n_d$ ). The general form of the decision function is:

$$\Lambda = \begin{cases} 1 & (n_p, n_d) \in \mathcal{L} \\ 0 & (n_p, n_d) \in \bar{\mathcal{L}} \end{cases}$$

In which  $\mathcal{L}$  (stands for “Liability”) and  $\bar{\mathcal{L}}$  (stands for “No Liability”) stand for two mutually exclusive sets of possible pairs of  $n_p$  and  $n_d$  (i.e.,  $\mathcal{L} \cap \bar{\mathcal{L}} = \emptyset$ , and  $\emptyset$  stands for the empty set), and  $\mathcal{L}$  and  $\bar{\mathcal{L}}$  together include all possible pairs of  $n_p$  and  $n_d$  (i.e.,  $\mathcal{L} \cup \bar{\mathcal{L}} = \mathbb{N} \times \mathbb{N}$ , and  $\mathbb{N} \times \mathbb{N}$  is the universal set of all non-negative integer pairs).  $\Lambda = 1$  means that the fact finder renders a decision for the plaintiff, and  $\Lambda = 0$  means that the fact finder renders a decision for the defendant.

According to this generally formatted standard of proof, every possible collection of evidence on the court results in one and only one decision. Now the question remaining is how to properly decide  $\mathcal{L}$  and  $\bar{\mathcal{L}}$ .

**Assumption 5’ (Assumption of The Legislator’s Optimization Problem):** The legislator follows the Neyman-Pearson Theory. That is to say, for the legislator, the optimal standard of proof should: (1) control the probability of type-I errors below a LOW level which is acceptable to the society (this level is called the society’s tolerability of type-I errors and denoted as  $\alpha$ ), and (2) minimize the probability of type-II errors with a given investigation budget.

The Neyman-Pearson Theory was firstly introduced by Jerzy Neyman and Egon Pearson in their 1933 paper<sup>32</sup>. This theory says that, the “best” (in statistics language, the “uniformly most powerful test”, or UMPT for short) minimizes the probability of type-II errors (failing to reject the incorrect null hypothesis) while maintaining the probability of type-I errors (rejecting the correct null hypothesis) below a certain threshold value (usually called the significance level of the test, conventionally denoted as  $\alpha$ ). It is important to emphasize that, in the Neyman-Pearson Theory, the two types of errors are not considered

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<sup>32</sup> Neyman, J. and E. Pearson (1933), On the Problem of the Most Efficient Tests of Statistical Hypotheses, Philosophical Transactions of the Royal Society of London. Series A, Vol. 231, pp. 289-337

equal. Particularly, type-I errors are considered more serious than type-II errors, and that is why the Neyman-Pearson Theory explicitly controls the probability of type-I errors rather than the probability of type-II errors. (In convention, the significance level  $\alpha$  is usually set at a very low level such as 5%. Under such a low significance level, the possible minimum level of type-II errors most likely is very high, such as 20%.)

Although the Neyman-Pearson Theory has been established as the convention of hypothesis testing for nearly a century, there are astonishingly very few justifications for it. One justification argues that this theory is quite similar to human being's psychological decision making process. For example, the idea of significance level is very similar to the psychological concept *absolute threshold*<sup>33</sup>. Since type-I errors are more serious to us than type-II errors, we certainly want to control them under a certain level at which we cannot even notice their existence. In psychological words, we want to control the probability of type-I errors below society's absolute threshold. Another justification may be more persuasive: on one hand, this theory is easy to understand and apply. On the other hand, there is no competing criterion to it<sup>34</sup>. As a result, this theory dominates the kingdom of hypothesis testing. When hypothesis testing becomes more and more important to the modern world, people develop a new thinking pattern in accordance with the Neyman-Pearson Theory. In other words, the justification of the Neyman-Pearson Theory follows a bottom-up pattern: its correctness is not the cause of its popularity but rather the result<sup>35</sup>.

Moreover, there are some specific jurisprudential and methodological reasons to apply the Neyman-Pearson Theory to describe the inquisitorial legislator's optimization problem.

First, the conventional optimization models on fact-finding essentially follows the utilitarian philosophy, which argues that different people's utility (or disutility) is measurable and additive. Therefore, it in fact implies that a type-I error can be perfectly compensated by reducing a certain amount of type-II errors. However, type-I errors and type-II errors are largely apples and oranges to society. Even in the famous statement known as the Blackstone Ratio, Sir Blackstone only argues that "[it] is better that ten guilty persons escape than that one innocent suffer", but he never implies that it is better that one innocent suffer than eleven guilty person escape. In fact, what the error-cost-ratio truly is, or even whether this ratio really exists, is under furious debate<sup>36</sup>, so it is hardly trustworthy that fact finders really think in terms of the error-cost-ratio in decision making. In addition, no matter in civil cases or in criminal cases, people always

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<sup>33</sup> The Absolute Threshold Theory states that people will pay absolutely no attention to a stimulus (including information and signal) until it reaches a certain threshold. See, for example, "Absolute Threshold." Gale Encyclopedia of Psychology. 2001. Retrieved January 18, 2015 from Encyclopedia.com: <http://www.encyclopedia.com/doc/1G2-3406000012.html>

<sup>34</sup> In fact, although the Neyman-Pearson Theory was criticized by statisticians for decades, at least up to now, there turns out to be no better criterion over it.

<sup>35</sup> For a more detailed discussion of this phenomenon, please see

<sup>36</sup> See Volokh, A. (1997), n Guilty Men, University of Pennsylvania Law Review, Vol. 146, Page 173-216 for a comprehensive discussion about this ratio in criminal law.

care type-I errors in advance of type-II errors<sup>37</sup>. Therefore, a threshold-based decision making mechanism focusing on type-I errors suggested by the Neyman-Pearson Theory is more conform to the reality<sup>38</sup> in both criminal cases and civil cases.

Second, compared to the conventional error-cost-ratio approach, the Neyman-Pearson Theory requires less parameters in modeling. This is straightforward: to apply the error-cost-ratio approach, we need at least two more parameters: one for type-I error costs and the other for type-II error costs; however, we just need to add the  $\alpha$  parameter under the Neyman-Pearson Theory.

**Assumption 6' (Parties' Strategies and incentives Assumption):** Assume that the value of  $\alpha$  is the common knowledge to the inquisitorial judge and both parties, and  $\theta$  is the common knowledge of both parties but not the judge. The plaintiff moves first to decide whether to file the lawsuit. If no lawsuit is filed, both parties get a payoff of 0. If a lawsuit is filed, the defendant can choose to settle with the plaintiff by paying the plaintiff  $B$ , or the case will go to trial. Once a lawsuit goes to trial, the judge takes the duty to investigate and gets a lump sum budget to produce a total amount of  $N_j$  pieces of evidence of which  $n_p$  pieces are for the plaintiff and  $n_d$  pieces are for the defendant.  $N_j = n_p + n_d$ . At the end of the trial, the judge makes a decision based on ALL evidence obtained. If the defendant loses the trial, she needs to pay the plaintiff  $B$ ; if the defendant wins the trial, she needs to pay nothing to the plaintiff.

Under the inquisitorial system, the prepaid investigation budget is usually compensated by the parties after the judge renders her decision. However, because the compensation arrangement does not affect the judge's decision making, we will not discuss it here. Instead, we will discuss it in the comparison part later.

## IV. 2. The Likelihood Ratio Test

In statistics, based on a representative random sample of the population, there are many different methods

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<sup>37</sup> It is already a common sense in criminal law that people concern type-I errors prior to type-II errors. In civil law, however, many scholars argue that both types of errors are equally troublesome. In fact, a lot of empirical researches have shown that people care about type-I errors ahead of type-II errors in civil law cases. For example, Zamir, E., and I. Rotiv (2012), Loss Aversion, Omission Bias, and the Standard of Proof in Civil Litigation, *The Journal of Legal Studies*, Vol. 41, Page 165-207 shows that people are less incline to give a verdict for the plaintiff than for the defendant. Zamir and Rotiv (2012) suggests that phenomenon is consistent with the "omission bias" in psychology which refers to "the tendency to judge harmful actions as worse, or less moral than equally harmful omissions (inactions) due to the fact that actions are more obvious than inactions." ([http://en.wikipedia.org/wiki/Omission\\_bias](http://en.wikipedia.org/wiki/Omission_bias)) Since a verdict for the plaintiff is an action rather than an omission, people are hence more aware of type-I errors than a type-II errors even in civil cases.

<sup>38</sup> In fact, the Neyman-Pearson Theory is already ubiquitously accepted in every field associated with statistical hypothesis testing, such as econometrics. What is more, some psychological researches have already shown that the Neyman-Pearson Theory as a robust cognitive process. For a brief list of references in this area, please see Gigerenzer, G. (1991), From Tools to Theories: A Heuristic of Discovery in Cognitive Psychology, *Psychological Review*, Vol. 98, No. 2, Page 256.

to test hypotheses like “ $\mathbf{H}_0: \theta \leq 0.5$  versus  $\mathbf{H}_1: \theta > 0.5$ ”. However, according to the criteria of the Neyman-Pearson Theory, there is one “best” test among all of these available tests. That is the likelihood ratio test. This statistical result, widely known as the Neyman-Pearson Lemma, is proved by Neyman and Pearson in their famous 1933 paper. Next, we will introduce the general structure of the likelihood ratio test based on which we will derive the optimal standard of proof under the inquisitorial system. Our discussion is generally based on the International Encyclopedia of Statistics<sup>39</sup>.

In a typical likelihood ratio test, the null hypothesis is “random variable  $x$  follows a distribution with a cdf  $F_0$  and a pdf  $f_0$ ”, and the alternative hypothesis is “ $x$  follows a distribution with a cdf  $F_1$  and a pdf  $f_1$ ”. In statistics language, the hypothesis to be tested is

$$\mathbf{H}_0: x \sim f_0 \text{ versus } \mathbf{H}_1: x \sim f_1$$

The likelihood ratio test is based on likelihood ratio. Suppose we have a random sample containing  $n$  observations of  $x$ , which is denoted as  $X = (x_1, x_2, \dots, x_n)$ . Then we can define the likelihood ratio as follows:

$$\lambda(X) = \frac{f_0(x_1)f_0(x_2) \cdots f_0(x_n)}{f_1(x_1)f_1(x_2) \cdots f_1(x_n)}$$

Consequently, we can define a test function (or decision function)  $\Lambda(X)$  as follows:

$$\Lambda(X) = \begin{cases} 1 & \lambda > k \\ \gamma & \lambda = k \\ 0 & \lambda < k \end{cases}$$

In which the values of  $\gamma$  and  $k$  are determined by the function  $E[\Lambda|x \sim f_0] = \alpha$  to ensure the probability of type-I errors to be no greater than the significance level  $\alpha$ .

As Neyman and Pearson (1933) shows, this test is the optimal test according to the Neyman-Pearson Theory. In statistics words, this test is the UMPT for this hypothesis testing problem.

In addition, the application scope of the Neyman-Pearson Lemma can be further extended if the *monotonic likelihood ratio property* holds<sup>40</sup>, which results in a corollary more relevant to our discussion than the

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<sup>39</sup> “Neyman-Pearson Lemma”, International Encyclopedia of Statistics, pp. 945-946.

<sup>40</sup> The monotonic likelihood ratio property under this specific circumstance is defined as follows: Suppose all possible distributions for random variable  $x$  have the same function form  $f(x, \theta)$  except that they have different values for parameter  $\theta$  in their distribution functions. If for any  $\theta_2 > \theta_1$ , the value of the following likelihood ratio function

$$\lambda(X) = \frac{f(x_1, \theta_2)f(x_2, \theta_2) \cdots f(x_n, \theta_2)}{f(x_1, \theta_1)f(x_2, \theta_1) \cdots f(x_n, \theta_1)}$$

is a nondecreasing function of a real valued statistic  $Y(X)$ , then we say that the distribution  $f(x, \theta)$ s have *monotonic likelihood ratio* in  $Y(X)$ .

Neyman-Pearson Lemma itself. The corollary is discussed as follows:

Consider the following hypothesis testing problem:

$$\mathbf{H}_0: x \sim f_0 \in \{f(x, \theta) | \theta \leq \theta_0\} \text{ versus } \mathbf{H}_1: x \sim f_1 \in \{f(x, \theta) | \theta > \theta_0\}$$

Which can be simplified as:

$$\mathbf{H}_0: \theta \leq \theta_0 \text{ versus } \mathbf{H}_1: \theta > \theta_0$$

If the distribution  $f(x, \theta)$ s have monotonic likelihood ratio in  $Y(X)$ , the UMPT for this testing problem can be given as follows<sup>41</sup>:

$$\Lambda(X) = \begin{cases} 1 & Y(X) > k \\ \gamma & Y(X) = k \\ 0 & Y(X) < k \end{cases}$$

In which the values of  $\gamma$  and  $k$  are determined by the equation  $E[\Lambda | \theta = \theta_0] = \alpha$  to ensure the probability of type-I errors to be no greater than the significance level  $\alpha$ .

## IV. 3 Deriving the Optimal Inquisitorial Standard of proof

As we have discussed again and again, the inquisitorial fact-finding problem is equivalent to the following hypothesis testing problem:

$$\mathbf{H}_0: \theta \leq 0.5 \text{ versus } \mathbf{H}_1: \theta > 0.5$$

Next, we will firstly derive the UMPT for this testing problem. Then we prove that the derived decision function satisfies the criteria defined in Assumption 4' and Assumption 5'.

**Proposition 2:** The optimal standard of proof under the inquisitorial system is:

$$\Lambda = \begin{cases} 1 & n_p > k \\ 0 & n_p \leq k \end{cases}$$

In which  $k = 0.5N_j + 0.5Z_{1-\alpha}\sqrt{N_j}$ , and  $Z_{1-\alpha}$  is defined by  $\Phi(Z_{1-\alpha}) = 1 - \alpha$  ( $\Phi$  stands for the cdf for standard normal distribution  $\mathcal{N}(0,1)$ ).

Proof: According to the discussions following Assumption 2, if the judge's investigations result in  $n_p$  pieces of evidence indicating liability out of all  $N_j$  pieces of evidence, the likelihood function will be  $\theta^{n_p}(1 - \theta)^{N_j - n_p}$ . Therefore, for any  $\theta_2 > \theta_1$  we have the following likelihood ratio function:

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<sup>41</sup> For a proof of this result, please see Shao, J. (2003), *Mathematical Statistics* (2<sup>nd</sup> Edition), Springer-Verlag, New York. pp. 399-400.

$$\lambda = \frac{\theta_2^{n_p} (1 - \theta_2)^{N_J - n_p}}{\theta_1^{n_p} (1 - \theta_1)^{N_J - n_p}}$$

Note that when  $\theta_2 > \theta_1$ ,  $\lambda$  is a monotonic increasing function with respect to  $n_p$ . Therefore, the distribution  $\mathcal{B}(N_J, \theta)$ s have monotonic likelihood ratio in  $n_p$ . According to the corollary of the Neyman-Pearson Lemma which we introduced at the end of Section IV. 2, we know that the UMPT for the hypothesis testing problem  $\mathbf{H}_0: \theta \leq 0.5$  versus  $\mathbf{H}_1: \theta > 0.5$  takes the following form:

$$\Lambda = \begin{cases} 1 & n_p > k \\ \gamma & n_p = k \\ 0 & n_p < k \end{cases}$$

In which  $k$  and  $\gamma$  satisfy  $E[\Lambda|\theta = 0.5] = \alpha$  which is equivalent to:

$$\text{Probability}(n_p > k) + \gamma \text{Probability}(n_p = k) = \alpha$$

Next we will derive the values of  $k$  and  $\gamma$ .

If  $\theta = 0.5$ ,  $n_p$  follows a binomial distribution  $\mathcal{B}(N_J, 0.5)$  which can be approximated by a normal distribution  $\mathcal{N}(0.5N_J, 0.25N_J)$ .<sup>42</sup> With this approximation, we have  $\text{Probability}(n_p = k) = 0$ , so the value of  $\gamma$  is irrelevant to our decision, and we can simply assign  $\gamma = 0$ . As a result,  $E[\Lambda|\theta = 0.5] = \alpha$  is now equivalent to  $\text{Probability}(n_p > k) = \alpha$ . Because the term  $\frac{n_p - 0.5N_J}{0.5\sqrt{N_J}}$  follows the standard normal distribution  $\mathcal{N}(0,1)$ , we have

$$\text{Probability}(n_p > k) = \text{Probability}\left(\frac{n_p - 0.5N_J}{0.5\sqrt{N_J}} > \frac{k - 0.5N_J}{0.5\sqrt{N_J}}\right) = 1 - \Phi\left(\frac{k - 0.5N_J}{0.5\sqrt{N_J}}\right) = \alpha$$

Which gives:

$$k = 0.5N_J + 0.5Z_{1-\alpha}\sqrt{N_J}$$

In which  $Z_{1-\alpha}$  is defined by  $\Phi(Z_{1-\alpha}) = 1 - \alpha$  ( $\Phi$  stands for the cumulative distribution function for standard normal distribution).

Therefore, the UMPT for the hypothesis testing problem  $\mathbf{H}_0: \theta \leq 0.5$  versus  $\mathbf{H}_1: \theta > 0.5$  is:

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<sup>42</sup> Again, here we follow Froeb and Kobayashi (1996)'s approach that approximates the discrete numbers with continuous numbers.

$$\Lambda = \begin{cases} 1 & n_p > k \\ 0 & n_p \leq k \end{cases}$$

In which  $k = 0.5N_j + 0.5Z_{1-\alpha}\sqrt{N_j}$ .

It is easy to verify that this standard of proof satisfy the criteria specified in Assumption 4' and 5'. On one hand, its format conforms to the general format stated in Assumption 4'. On the other hand, because this decision function is the UMPT for the hypothesis testing problem which is equivalent to our fact-finding problem, it by definition satisfies the Neyman-Pearson Theory and thus Assumption 5'. As a result, this decision function, or more relevantly this standard of proof, is optimal under the inquisitorial system<sup>43</sup>.

The table follows gives some example values of  $k$  based on different values of  $N_j$  and  $\alpha$ .

**Table 1 Standard of Proof  $k$  based on Different Values of  $N_j$  and  $\alpha$**

$\alpha \backslash N_j$	5	10	15	20	25	30
<b>0.50</b>	2.5	5	7.5	10	12.5	15
<b>0.45</b>	2.64	5.20	7.74	10.28	12.81	15.34
<b>0.40</b>	2.78	5.40	7.99	10.57	13.13	15.69
<b>0.35</b>	2.93	5.61	8.25	10.86	13.46	16.06
<b>0.30</b>	3.09	5.83	8.52	11.17	13.81	16.44
<b>0.25</b>	3.25	6.07	8.81	11.51	14.19	16.85
<b>0.20</b>	3.44	6.33	9.13	11.88	14.60	17.30
<b>0.15</b>	3.66	6.64	9.51	12.32	15.09	17.84
<b>0.10</b>	3.93	7.03	9.98	12.87	15.70	18.51
<b>0.05</b>	4.34	7.60	10.69	13.68	16.61	19.50

Several intuitions can be derived from the above stated optimal standard of proof under the inquisitorial system:

Frist of all,  $k$  is an increasing function with respect to  $Z_{1-\alpha}$  which is further a decreasing function with respect to  $\alpha$ . As a result,  $k$  is decreasing with respect to  $\alpha$  which stands for the society's tolerability of type-I errors. Intuitively, the optimal standard of proof under the inquisitorial system is decreasing with the society's tolerability of type-I errors. This is easy to understand and exactly in accordance with the conventional wisdom: when the society cannot tolerate a lot of type-I errors, it is desirable to raise the standard of proof to make those truly not liable defendants less likely to be found liable. On the other hand, if the society can tolerate a lot of type-I errors, a lower standard of proof is desired in order to have more

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<sup>43</sup> Note that the key of the decision function (or the standard of proof) is the threshold value  $k$ , so we can also call the threshold value  $k$  the standard of proof in a narrow sense.

truly liable defendants finally found liable in trials.

Second, it is easy to see that when  $\alpha < 0.5$ , the standard of proof ( $k$ ) is increasing with respect to the total number of evidence presented to the court ( $N_j$ ). However, the second order derivative of  $k$  with respect to  $N_j$  is negative. Therefore, the standard of proof does not grow proportionally with  $N_j$ . Take some values of  $k$  in Table 1 to make an example: When  $\alpha = 0.3$  and  $N_j = 15$ , we need at least 8.52 pieces of evidence indicating liability to make a decision for the plaintiff. However, if another 15 pieces of evidence become available to the court, we do not need another 8.52 pieces evidence indicating liability out of the new 15 pieces to make a decision for the plaintiff. Instead, we just need another 7.92 pieces out of the new 15 pieces of evidence to render a decision for the plaintiff. Intuitively, as the society spends more on investigation, the information about the defendant's "true type" would be more complete and reliable, so the judges can make decisions more "aggressively" rather than "conservatively".

## V. Comparing the Two Systems

### V. 1. Truth Revealing and Dispute Resolving: The Two Evaluation Angles

Truth revealing and dispute resolving are the two most frequently referred criteria when comparing the adversarial system and the inquisitorial system. However, there is not yet a consensus about which system is superior over the other according to either criterion. Following provides a general review of relevant discussions.

- a. Adversarial System in Truth Revealing. There are many arguments supporting the adversarial system as a better truth revealing system. First of all, the form of the adversarial trial is similar to some methods searching the truth in academia, such as the philosophical discourse. Philosophers as early as Socrates and Plato believe that proposing and disputing ideas, just as described in the famous book *The Dialogues*, is an efficient way to end up with the truth. However, opponents of the adversarial system argue that, in academic debates, the disputing scholars in fact collaborate with each other for the same goal of revealing the truth; however, in legal trials, the parties dispute purely to beat their opponents. Therefore, the adversarial system does not reveal the truth as efficiently as the academic discourses do. Second, the proponents of the adversarial system argue that the adversarial system encourages the parties to provide more information to the court than the inquisitorial system does, and with more information the court is more likely to arrive in a correct decision. However, the opponents argue that more information does not necessarily arrive in more correct decision, because the information can be manipulated or even distorted by parties for their own interests.
- b. Inquisitorial System in Truth Revealing. The proponents of the inquisitorial system argue that under the inquisitorial system, the judge has direct access to unbiased information which is critical for making the correct decision. However, the opponents argue that the judges may lack the incentive to

collect enough information, and the inquisitorial adjudication process is more vulnerable to corruption than its adversarial counterpart.

- c. Adversarial System in Dispute Resolving. According to a lot of legal historians, the adversarial system was firstly introduced to replace duel which was a common way of resolving disputes during the medieval time. That is to say, the very original intention of the adversarial system is exactly resolving disputes. In addition, just as the stronger person survives from the duel, the person who values the case more is more likely to prevail under the adversarial system. This is because the person who values the case more is more willing to spend more on the litigation and thus is more likely to win the case<sup>44</sup>. Therefore, the adversarial system not only is designed to resolve disputes, but also resolves disputes in an efficient way.
- d. Inquisitorial System in Dispute Resolving. Compared to the adversarial system, the dispute-resolving aspect of the inquisitorial system is not extensively discussed. A justification for the inquisitorial system's superiority in resolving disputes is that, the inquisitorial trial is a long time, continuous process, so during the inquisitorial hearing, the judge has more opportunities to suggest possible resolutions of the dispute to the parties<sup>45</sup>. Therefore, the inquisitorial system sometimes can provide early resolution for disputes and hence avoid further wastes of litigation resources.

## V. 2. Comparing Inquisitorial and Adversarial Truth Revealing

In this section, we will compare the two legal systems together with their standards of proof from various angles. Our comparison proceeds as follows: First, we compare the accuracy of the two systems with respect to different levels of social investigation costs. Second, we compare the accuracy of the adversarial system with the accuracy of an inquisitorial system applying a uniform investigation budget. Third, we compare the two systems from the angle of dispute-resolving.

To make the comparison more straightforward and “fair”, we assume that society has the theoretical maximum tolerance of type-I errors. That is to say, society accepts the fact-finding system as long as its accuracy on type-I errors is better than “tossing a fair coin”. Mathematically, the society's tolerability parameter  $\alpha = 0.5$ . As a result, the optimal standard of proof under the inquisitorial system turns out to be:

$$\Lambda = \begin{cases} 1 & n_p > 0.5N_j \\ 0 & n_p \leq 0.5N_j \end{cases}$$

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<sup>44</sup> For a formal theoretical analysis of this statement, please see Goodman, John C. (1978) An Economic Theory of the Evolution of Common Law, *The Journal of Legal Studies*, 7(2), pp. 393-406.

<sup>45</sup> See generally in Langbein, John H. (1985) The German Advantage in Civil Procedure. *The University of Chicago Law Review*, 52(4): Page 828.

Which is in the form equivalent to the preponderance of the evidence standard under the adversarial system. Now we will compare these two systems with these in-the-same-form standards of proof, and the following conclusions can be derived:

**Conclusion 1: The accuracy of the adversarial system does not improve with society's overall investigation efforts, but the accuracy of the inquisitorial system does.**

From Corollary 1-2, we know that the probability of adjudication errors does not vary with the disputed value ( $B$ ). On the other hand, however, society's overall investigation efforts is an increasing function with respect to  $B$  as given below:

$$c = \begin{cases} \frac{\theta}{1-\theta}B, \theta \leq 0.5 \\ \frac{1-\theta}{\theta}B, \theta > 0.5 \end{cases}$$

That is to say, given the strength of the case, more investigation efforts under the adversarial system does not improve the accuracy of the legal system.

Now let us consider the probability of adjudication errors under the inquisitorial system when  $\alpha = 0.5$ .

As we discussed previously, when  $\alpha = 0.5$ , the optimal standard of proof under the inquisitorial system takes the following form:

$$\Lambda = \begin{cases} 1 & n_p > 0.5N_j \\ 0 & n_p \leq 0.5N_j \end{cases}$$

When  $\theta \leq 0.5$ , adjudication errors are type-I errors, and the probability can be denoted as:

$$\text{Prob}(\Lambda = 1 | \theta \leq 0.5) = \text{Prob}(n_p > 0.5N_j | \theta \leq 0.5)$$

With the normal approximation, we have  $n_p \sim \mathcal{N}(\theta N_j, \theta(1-\theta)N_j)$ . Therefore,

$$\begin{aligned} \text{Prob}(n_p > 0.5N_j | \theta \leq 0.5) &= \text{Prob}\left(\frac{n_p - \theta N_j}{\sqrt{\theta(1-\theta)N_j}} > \frac{0.5N_j - \theta N_j}{\sqrt{\theta(1-\theta)N_j}} \mid \theta \leq 0.5\right) \\ &= 1 - \Phi\left(\frac{0.5 - \theta}{\sqrt{\theta(1-\theta)}}\sqrt{N_j}\right) \quad \theta \leq 0.5 \end{aligned}$$

With the same reasoning, we can develop the probability of type-II errors to be:

$$1 - \Phi\left(\frac{\theta - 0.5}{\sqrt{\theta(1-\theta)}}\sqrt{N_j}\right) \quad \theta > 0.5$$

It is easy to note that these probabilities are decreasing with respect to the value of  $N_j$ . Therefore, the

accuracy of the inquisitorial system improves with the society's overall investigation efforts.

**Conclusion 2: The inquisitorial authority can produce LESS adjudication errors by applying a uniform reasonable investigation budget to all cases.**

Next we will compare the two systems from a practical perspective. We assume that the inquisitorial system applies a uniform investigation budget to all cases because the judge does not know the true value of  $\theta$ . Our argument is, if the inquisitorial system can incur less adjudication errors than the adversarial system by applying a reasonably low uniform investigation budget, we can say that the inquisitorial system is more accurate than the adversarial system, because any higher budget level can only make the system even more accurate than the adversarial system as we have shown in the last section.

Again, let us consider the situation when  $\alpha = 0.5$ . The optimal standard of proof under the inquisitorial system takes the following form:

$$\Lambda = \begin{cases} 1 & n_p > 0.5N_j \\ 0 & n_p \leq 0.5N_j \end{cases}$$

The probability for type-I errors is:

$$1 - \Phi\left(\frac{0.5 - \theta}{\sqrt{\theta(1 - \theta)}}\sqrt{N_j}\right) \quad \theta \leq 0.5$$

The probability for type-II errors is:

$$1 - \Phi\left(\frac{\theta - 0.5}{\sqrt{\theta(1 - \theta)}}\sqrt{N_j}\right) \quad \theta > 0.5$$

Again, it is easy to note that these probabilities are decreasing with respect to  $N_j$ . As a result, we also just need to find the  $N_j$  under which the probability of adjudication errors under the inquisitorial system is obviously below that under the adversarial system. Then the same conclusion can be applied to all  $N_j$ 's with higher values.

In the follows we present the comparison plots for  $N_j = 3, 5, 7$ , and it is obvious to see that when  $N_j = 7$ , the inquisitorial system is strictly more accurate than the adversarial system for all cases. Intuitively, simply by producing 7 pieces of evidence for any case and making the decision accordingly, the inquisitorial system can avoid more adjudication errors than the adversarial system.

Figure 2 A Comparison of Adjudication Errors when  $N_J = 3$

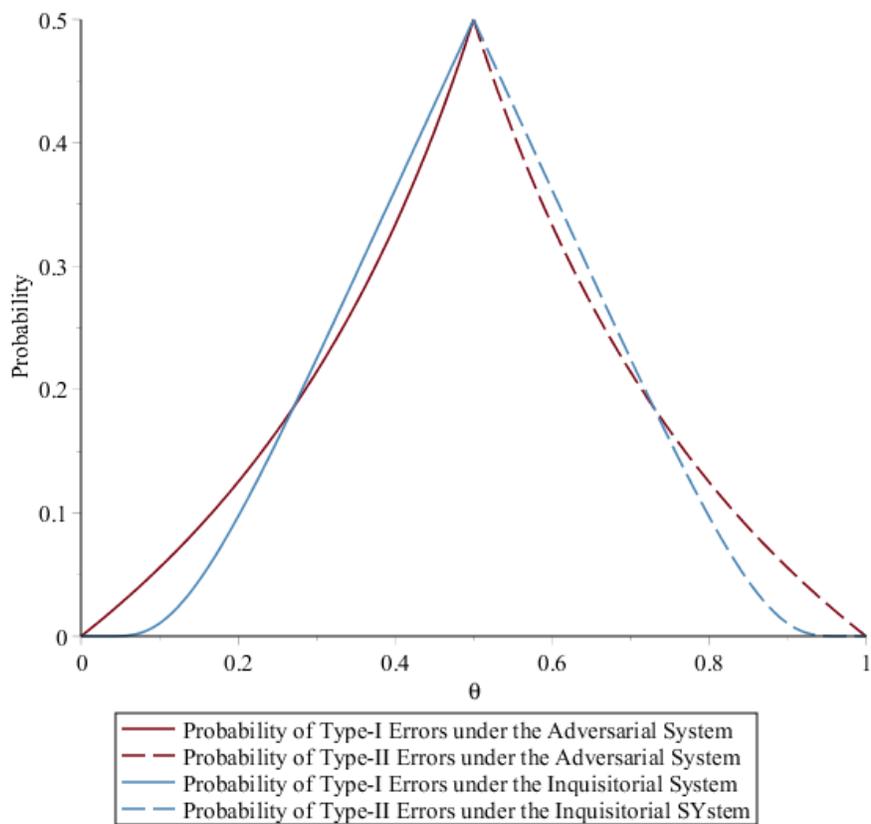


Figure 3 A Comparison of Adjudication Errors when  $N_J = 5$

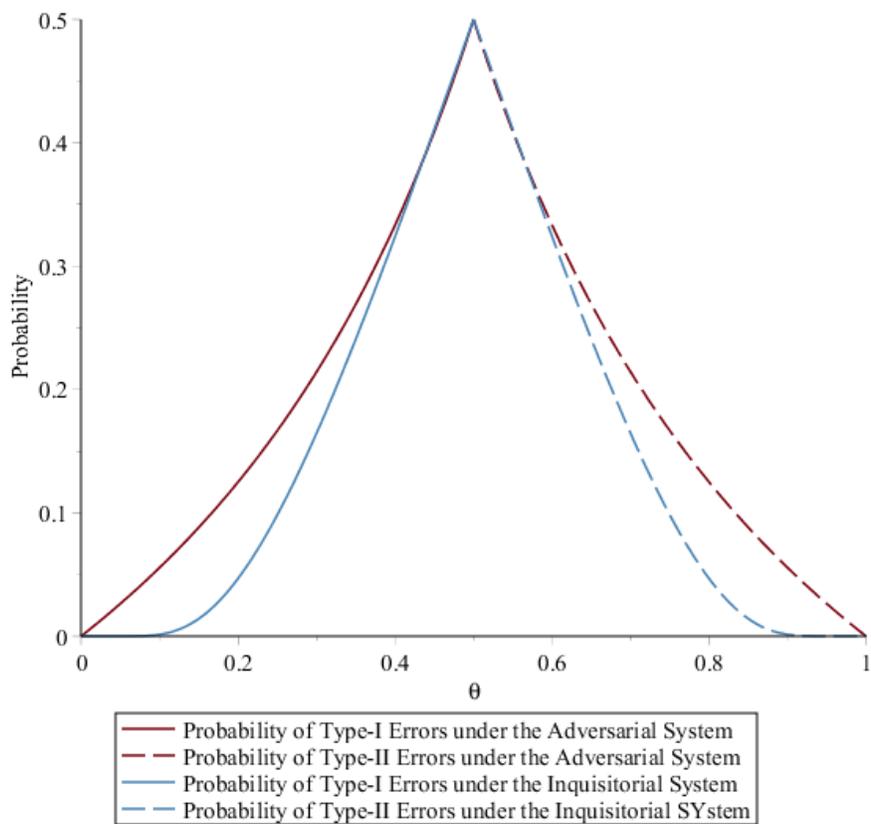
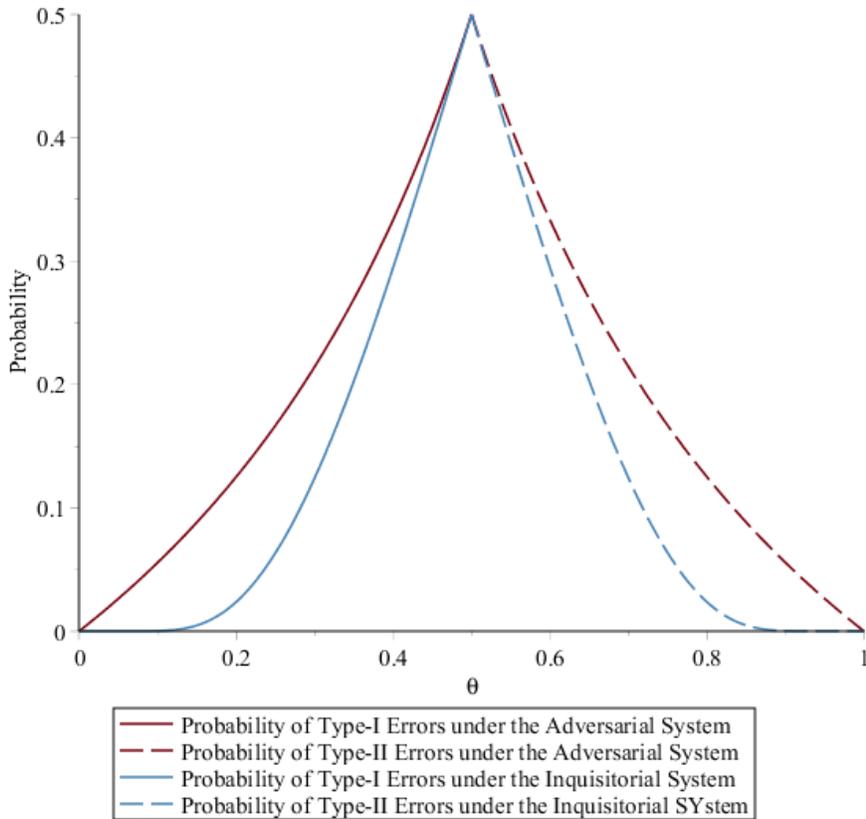


Figure 4 A Comparison of Adjudication Errors when  $N_J = 7$



**Conclusion 3: The advantage of the adversarial system is that, under the adversarial system, the party not backed by the truth always finds it not profitable to go to trial. Therefore it discourages frivolous litigations and encourages settlements. Therefore, it is more frugal in resolving disputes by saving considerable litigation resources.**

Up to now, we have not considered the litigation cost arrangement under the inquisitorial system which influences the plaintiff’s decision of filing the case. In fact, when a uniform investigation budget is applied, it is easy to see that there are always some plaintiffs who strictly prefer filing frivolous cases and some truly liable defendants who always refuse to settle.

To see this point, let us think about the “loser pays” rule which is the least friendly regime to the party not backed by the truth. Namely, the inquisitorial authority applies a uniform investigation budget to all cases, and after the trial, the loser is ordered to compensate all litigation costs incurred. Under this regime, because the probabilities of adjudication errors is fixed, a frivolous plaintiff may still find it profitable to file the case if the disputed value is high enough. For example, Let us look into Figure 4 in which the inquisitorial authority applies a budget of 7. If the case strength parameter is 0.4, the frivolous plaintiff’s probability to win the case is about 30%. The plaintiff files the case if  $0.3 * B - 0.7 * 7 > 0$ . In other words, if the disputed value is higher than 16.3 ( $0.7 \times 7 \div 0.3$ ), it is always profitable to file the frivolous

lawsuit. Therefore, there is always some frivolous litigations under such regime. By the same reasoning, we can prove that there is always some truly liable defendants who refuses to settle.

On the other hand, as we have discussed in corollary 2-4 and 2-5, under the adversarial system, the party not backed by the truth will never find it profitable to go to trial: The frivolous plaintiffs will choose not to file, while the truly liable defendants will always choose to settle. As a result, there will be less litigation under the adversarial system than under the inquisitorial system, and a considerable amount of litigation resources can be saved.

## **VI. Summary**

A prominent scholar in comparative law study, Mirjan Damaska, stated in his famous book *The Faces of Justice and State Authority: A Comparative Approach to the Legal Process* that the common law system is a system only to resolve disputes with whatever methods, whereas the civil law system is a system which resolves disputes based on the possible best reconstruction of the truth. This paper compares these two systems from an entry point of evidence presenting and interpreting, and our conclusions greatly coincide with Damaska's statement: on one hand, in most circumstances, the adversarial fact-finding is both wasteful and inaccurate compared to the inquisitorial fact-finding; on the other hand, the advantage of the adversarial system is that it sends the party not backed by the truth a clear signal that "it is not profitable for you to litigate", which simultaneously discourages frivolous litigations and encourages settlements.

Our paper also sheds light on the fusion of different legal systems which has become a global trend since the past century. On one hand, the common law systems, which are traditionally short of accuracy, have imported considerable inquisitorial features to compensate. One of the most notable examples is the introduction of the discovery process to the U.S. legal system. On the other hand, the civil law systems have greatly weakened the inquisitorial role of judges in areas where the truth is not so important relatively to an efficient resolution of the dispute. For example, as many commenters have observed, the French civil courts now rarely execute their inquisitorial powers, even if those powers are explicitly guaranteed by the French Code of Civil Procedure.

# Appendix

## Proof of Proposition 1

Firstly, we will show that there is no pure strategy equilibrium for the evidence presentation game. This is easy to see. Suppose there was a pure strategy equilibrium denoted as  $(n_p^*, n_d^*)$  for the evidence presentation game.

a). If  $n_p^* > n_d^* > 0$ , the plaintiff always wins the case. However, for the defendant, now she would like to deviate from this equilibrium. Specifically, she could either choose to present no evidence at all to save the “wasted” litigation costs (as a result, she can get better off by  $\frac{n_d^*}{1-\theta}$ ) or choose to present more evidence to win the case (she needs to present another  $n_p^* - n_d^*$  pieces of evidence to win the case, and she can get better off by  $\max\{0, B - \frac{n_p^* - n_d^*}{1-\theta}\}$ <sup>46</sup>). Putting everything together, if  $B - \frac{n_p^* - n_d^*}{1-\theta} > \frac{n_d^*}{1-\theta}$ , the defendant would present more evidence to win the case; if  $B - \frac{n_p^* - n_d^*}{1-\theta} < \frac{n_d^*}{1-\theta}$ , the defendant would like to present no evidence at all and save the wasted litigation costs. In a word, the defendant would always like to deviate from the hypothetical equilibrium.

b). If  $n_p^* > n_d^* = 0$ , the plaintiff always wins the case. However, the plaintiff could get better off if he reduces his presentation as long as  $n_p > n_d^*$  still holds. For example, she could choose  $n_p^{**} = \frac{n_p^* + n_d^*}{2}$  with which she could save her litigation costs without losing the case. Therefore, the plaintiff would always like to deviate from this hypothetical equilibrium.

c). If  $n_d^* \geq n_p^* > 0$ , the defendant always wins the case. With similar reasoning as in a), we can prove that the plaintiff would always like to deviate from the hypothetical equilibrium.

d). If  $n_d^* > n_p^* = 0$ , the defendant always wins the case. With similar reasoning in b), we can prove that the defendant would always like to deviate from the hypothetical equilibrium.

e). If  $n_d^* = n_p^* = 0$ , the defendant always wins the case. However, the plaintiff could get better off by present a tiny amount of evidence to win the case. Therefore, the plaintiff would always like to deviate from the hypothetical equilibrium.

Putting a) to e) together, we can see that any possible pair of  $(n_p^*, n_d^*)$  is not an equilibrium for the

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<sup>46</sup> If  $B - \frac{n_p^* - n_d^*}{1-\theta} < 0$ , i.e., the money saved from winning the case cannot fully compensate the extra litigation cost, it is not rational for the defendant to invest more in evidence to win the case.

evidence presentation game. Therefore we prove that there is no pure strategy equilibrium for the game<sup>47</sup>.

Next, we will derive the mixed strategy equilibrium for the game.

Suppose the plaintiff randomizes  $n_p$  according to a cdf  $F_p(x)$  (and the corresponding pdf is  $f_p(x)$ ), and suppose the defendant randomizes  $n_d$  according to a cdf  $F_d(x)$  (and the corresponding pdf is  $f_d(x)$ ).

Let us first derive the equilibrium for  $\theta \leq 0.5$ . When  $\theta \leq 0.5$ , the support for both  $F_p(x)$  and  $F_d(x)$  is  $[0, \theta B]$ . This is easy to see, because it is not rational for the plaintiff to present  $n_p$  such that  $\frac{n_p}{\theta} > B$  or equivalently  $n_p > \theta B$  (in such case, her expected payoff from litigation will be strictly negative regardless of the litigation result). Similarly, it is not rational for the plaintiff to present  $n_d$  such that  $\frac{n_d}{1-\theta} > B$  or equivalently  $n_p > (1-\theta)B$ . In addition, because  $\theta \leq 0.5$ , we have  $(1-\theta)B \geq \theta B$ .

Because  $n_p \leq \theta B$ , it is irrational for the defendant to present any number larger than  $\theta B$ , as she can win any case by presenting  $n_d = \theta B$  according to the standard of preponderance of the evidence.

If the defendant follows this strategy and the plaintiff chooses  $n_p$ , the expected payoff of the plaintiff from litigation will be:

$$u_p(n_p) = \int_0^{n_p} B dF_d(n_d) + \int_{n_p}^{\theta B} 0 dF_d(n_d) - \frac{n_p}{\theta} = BF_d(n_p) - \frac{n_p}{\theta}$$

To randomize, the plaintiff should be indifferent between any  $n_p$  in  $[0, \theta B]$ . That is to say, the value of  $u_p(n_p)$  is constant for any  $n_p \in [0, \theta B]$ . That is to say:

$$u_p(n_p) = BF_d(n_p) - \frac{n_p}{\theta} = \text{constant}$$

First order differentiate the entire equation, we have:

$$\frac{\partial u_p(n_p)}{\partial n_p} = f_d(n_p) - \frac{1}{\theta B} = 0, n_p \in [0, \theta B]$$

which is equivalent to:

$$f_d(n_p) = \frac{1}{\theta B}, n_p \in [0, \theta B]$$

Change  $n_p$  in the equation above to  $x$ , we have:

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<sup>47</sup> It may be noticed that this evidence presentation game is quite similar to the Dollar Auction game which has been extensively studied by game theorists.

$$f_d(x) = \frac{1}{\theta B}, x \in [0, \theta B]$$

Note that  $\int_0^{\theta B} f_d(x)dx = 1$ , so there is no mass point for  $F_d(x)$ , and  $F_d(x) = \frac{x}{\theta B}, x \in [0, \theta B]$

On the other hand, if the plaintiff follows this strategy and the defendant chooses  $n_d$ , the defendant's expected payoff will be:

$$u_d(n_d) = \int_0^{n_d} 0dF_p(n_p) + \int_{n_d}^{\theta B} (-B)dF_p(n_p) - \frac{n_d}{1-\theta} = -B(1 - F_p(n_d)) - \frac{n_d}{1-\theta}$$

With the same reasoning we applied before, we have:

$$\frac{\partial u_d(n_d)}{\partial n_d} = Bf_p(n_d) - \frac{1}{(1-\theta)}, n_d \in [0, \theta B]$$

Which is equivalent to:

$$f_p(n_d) = \frac{1}{(1-\theta)B}, n_d \in [0, \theta B]$$

Change  $n_d$  in the equation above to  $x$ , we have:

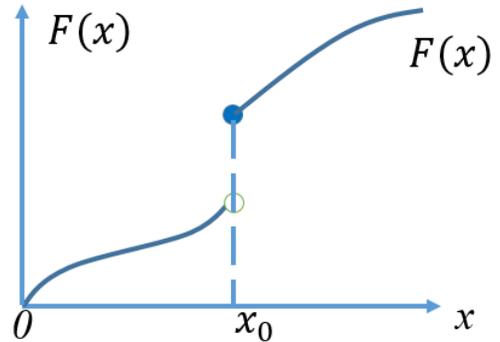
$$f_p(x) = \frac{1}{(1-\theta)B}, x \in [0, \theta B]$$

Note that  $\int_0^{\theta B} f_p(x)dx < 1$ , so there is a mass point for  $F_p(x)$ . Because  $f_p(x) = \frac{1}{(1-\theta)B} < \infty$ , the mass point can only be  $x = 0$ . For example, if the mass point was  $x_0 \in (0, \theta B)$ , we would have  $\text{Prob}(x = x_0) > 0$ , which means

$$\text{Prob}(x = x_0) = F(x_0) - F(x_0^-) > 0$$

Graphically, the cdf function  $F(x)$  "jumps" at the value  $x_0$  as shown on the right:

In this case, the function  $F(x)$  is not differentiable at the point  $x_0$ , or  $f_p(x) = \infty$ . Therefore, the mass point cannot be in  $(0, \frac{\theta B}{c})$ .



As a result,  $F_p(0) = 1 - \int_0^{\theta B} f_p(x)dx = 1 - \frac{\theta}{(1-\theta)}$ .

Therefore, we have  $F_p(x) = 1 - \frac{\theta}{(1-\theta)} + \frac{x}{(1-\theta)B}$ ,  $x \in [0, \theta B]$ , and the first half of the equilibrium is proved.

Symmetrically, we can prove the second half of the equilibrium (the case of  $\theta > 0.5$ ) with the same reasoning, and the second half of the equilibrium is given as follows:

Based on the equilibrium, the probability for the plaintiff to win the case conditioning on that the defendant presents  $n_d$  pieces of evidence is:

$$1 - F_p(n_d) = \begin{cases} \frac{\theta B - n_d}{(1-\theta)B} & n_d \in [0, \theta B] \text{ and } \theta \leq 0.5 \\ \frac{(1-\theta)B - n_d}{(1-\theta)B} & n_d \in [0, (1-\theta)B] \text{ and } \theta > 0.5 \end{cases}$$

Therefore, the unconditional probability for the plaintiff to win the case is:

$$\mathcal{P}_p = \begin{cases} \int_0^{\theta B} \frac{\theta B - n_d}{(1-\theta)B} dF_d(n_d) & \theta \leq 0.5 \\ \int_0^{(1-\theta)B} \frac{(1-\theta)B - n_d}{(1-\theta)B} dF_d(n_d) & \theta > 0.5 \end{cases}$$

Substitute the defendant's strategy  $F_d(n_d)$  into the integrations above, it is straightforward to get the unconditional probability for the plaintiff to win the case is:

$$\mathcal{P}_p = \begin{cases} \frac{1}{2} \cdot \frac{\theta}{(1-\theta)} & \theta \leq 0.5 \\ 1 - \frac{1}{2} \cdot \frac{(1-\theta)}{\theta} & \theta > 0.5 \end{cases}$$

When the plaintiff decides to present  $n_p$  pieces of evidence, her expected litigation cost is  $\frac{n_p}{\theta}$ . Therefore, her unconditional litigation cost is:

$$\mathcal{C}_p = \begin{cases} \int_0^{\theta B} \frac{n_p}{\theta} dF_p(n_p) & \theta \leq 0.5 \\ \int_0^{(1-\theta)B} \frac{n_p}{\theta} dF_p(n_p) & \theta > 0.5 \end{cases}$$

Substitute the plaintiff's strategy  $F_p(n_p)$  into the integrations above, we have:

$$C_p = \begin{cases} \frac{1}{2} \cdot \frac{\theta}{(1-\theta)} B & \theta \leq 0.5 \\ \frac{1}{2} \cdot \frac{(1-\theta)}{\theta} B & \theta > 0.5 \end{cases}$$

We know that  $\Pi_p = \mathcal{P}_p \cdot B - C_p$ . Substitute  $\mathcal{P}_p$  and  $C_p$  which we derived above, and it is straightforward to get:

$$\Pi_p = \begin{cases} 0, & \theta \leq 0.5 \\ B - \frac{(1-\theta)}{\theta} B, & \theta > 0.5 \end{cases}$$

Similarly, we can derive the value of  $\Pi_d$ .