The Welfare Costs of Skill-Mismatch Employment*

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Abstract

Skill-mismatch employment occurs when high-skilled individuals accept employment in jobs for which they are over-qualified, potentially crowding out the welfare of low-skilled individuals. We study the aggregate welfare implications of this phenomenon by developing a tractable general equilibrium model featuring skill-mismatch employment and on-the-job search. We derive a set of efficiency conditions that describe the labor market distortions associated with these two model features and illustrate how they alter the standard notion of the labor wedges inherent in general equilibrium search models. In a quantitative exercise, we calibrate the model to U.S. data and show that the distortions associated with skill-mismatch employment are distributional and quantitatively significant. Our results show that high-skilled individuals gain at the expense of the low-skilled.

Keywords: Job-to-job transitions, labor market frictions, crowding out
JEL Classification: E24, J31, J64

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1 Introduction

Empirical evidence suggests that labor market mismatch may have become more prevalent in recent years amid labor market slackness resulting from the global recession.[1] The growth of mismatch employment has important policy implications. For example, existing research suggests that skill-mismatch employment, which occurs when high-skilled individuals accept employment in jobs for which they are over-qualified, can lead to long-lasting scarring effects.[2] In addition, there may be externalities that arise as the search behavior of high-skilled job seekers crowds out that of low-skilled job seekers. This crowding out could prolong the recovery of the labor market, particularly for low-skilled individuals.

In this paper, we develop a model to better understand the labor market distortions associated with skill-mismatch. Our framework builds on the seminal work of Diamond (1982), Mortensen and Pissarides (1994), and Pissarides (2000) by introducing two-sided heterogeneity. Low-skilled individuals are qualified only for low-tech jobs. In contrast, high-skilled individuals are qualified for employment in both high- and low-tech jobs. Skill-mismatch is defined as a situation in which a high-skilled job seeker accepts a position with a low-tech firm. As in Dolado, Jansen, and Jimeno (2009), mismatch is "transitory" when on-the-job (OTJ) search by high-skilled workers can lead to job-to-job (JTJ) transitions out of mismatch employment and into the higher paying high-tech industry. When OTJ search is shut down in the model, mismatch is "permanent" in the sense that once formed a mismatch relationship is only terminated through job destruction.

Within this framework, our model captures the trade-off that high-skilled individuals face between accepting a lower quality job in order to move out of unemployment, but doing so at the cost of having to accept a lower wage in a job for which they are over-qualified. Moreover, the model reveals an externality whereby search activity of high-skilled individuals spills over to influence the labor market outcomes of low-skilled individuals. From the perspective of low-skilled individuals the welfare effects of this spillover could, in principle, be either positive or negative. Greater participation by high-skilled workers in the market for low-tech jobs could raise average match quality in the low-tech industry. As a result low-tech

[1] See, for example, Estevão and Tsounta (2012) and Şahin, Song, Topa, and Violante (2011).

[2] See, for example, Kahn (2010) and Oreopoulos, van Wachter, and Heisz (2012).

vacancy-posting incentives could rise as well, leading to an improvement in low-tech job-finding prospects that "crowds in" low-skilled welfare. Alternatively, increased participation by high-skilled workers in the market for low-tech jobs could result in inefficient substitution away from low-skilled labor, thereby "crowding out" low-skill welfare. Ultimately, which effect dominates is a quantitative question that we investigate in a calibrated version of the model.

The paper makes two contributions. First, we derive a set of efficiency conditions that provide a full analytic characterization of the distortions generated by both transitory and permanent mismatch. In this sense, we extend the results in Arseneau and Chugh (2012) to a more general setting with mismatch and OTJ search. In our framework labor market efficiency is described by a set of two static and three dynamic efficiency conditions. We show analytically that permanent mismatch distorts the labor market even in absence of unemployment benefits and the standard congestion externality owing to deviations from the Hosios condition. This distortion is amplified when mismatch is made temporary through the introduction of OTJ search.

The second contribution is to measure the quantitative magnitude of these various distortions in a carefully calibrated version of the model. We make use of publicly available data from the Bureau of Labor Statistics (BLS) on educational attainment to calibrate worker heterogeneity and BLS data on employment and wages by occupation to calibrate firm heterogeneity. Our calibration is consistent with a wide set of empirical labor market facts both at the aggregate as well as the disaggregate level. For example, among other things, the calibration captures an empirically realistic skill premium in the wage distribution and it endogenously gives rise to a fraction of employed individuals actively engaged in OTJ search that is in line with empirical estimates by Fallick and Fleischman (2005).

Our quantitative results show that the welfare effects of mismatch are distributional. For high-skilled households, permanent mismatch generates welfare gains of over 0.1 percent in terms of consumption equivalence, but these gains come exclusively at the expense of low-skilled households. Introducing OTJ search amplifies the welfare effects for both households, but this amplification is particularly pronounced for the low-skilled. Our results show that the transitory component of mismatch triples the welfare gains for high-skilled households to just under 0.5 percent of steady state consumption and raises the welfare costs for low-skilled households roughly ten-fold to 1.4 percent of steady state consumption. From a policy perspective, one conclusion to take is that the primary concern regarding mismatch manifests as a transitory issue, as opposed to a longer-lasting structural labor market issue.

Additionally, we conduct a sensitivity analysis to shed light on how mismatch activity influences distributional welfare effects. We find that, qualitatively, our result that low-skilled workers endure welfare costs as a result of mismatch is robust provided the model is parameterized so that mismatch workers earn a wage premium over low-skilled workers. Such a parameterization is supported empirically by Sicherman (1991). The sensitivity analysis also reveals that under any reasonable calibration, incremental mismatch activity always crowds out the welfare of low-skill individuals, even as average match quality in the low-tech sector improves.

In terms of related literature, our paper complements recent research examining other notions of mismatch, such as Shimer (2007) and Şahin et al. (2014). Our model of skill-mismatch builds on a strand of the labor search and matching literature that studies the impact of OTJ search on wages, unemployment, and vacancies.[3] Moreover, our focus on skill mismatch with two-sided heterogeneity ties our paper to Albrecht and Vroman (2002), Gautier (2002), Dolado, Jansen, and Jimeno (2009), Khalifa (2010), and Chassamboulli (2011), which are representative of a literature that studies the impact of two-sided heterogeneity on differences in wages, employment levels, and the persistence of unemployment rates across skill groups.[4] Menzio and Shi (2011) study business cycle fluctuations in a model with richer heterogeneity, but in their setup on-the-job search is efficient, whereas in ours it is not. Finally, some authors have studied efficiency in models with mismatch and OTJ search, as is done here, but these analyses are limited to a partial equilibrium view of the labor market.[5]

The remainder of the paper is organized as follows. The next section presents the model. Section 3 describes the socially efficient outcome. Section 4 compares the private equilibrium to the socially efficient equilibrium, allowing us to fully characterize the mismatch distortion.

[3] See Pissarides (1994), Shimer (2003, 2006), Nagypal (2005), and Moscarini (2005), among others.

[4] Krause and Lubik (2006) and Pries (2008) study similar issues with one-sided heterogeneity, while Epstein (2012) considers the effect of two-sided heterogeneity for the propagation of shocks.

[5] For example, Gautier, Tuelings, and van Vuuren (2010), and Albrecht, Navarro, and Vroman (2010).

Then, Section 5 uses a calibrated version of the model to produce a quantitative measure of the welfare costs of skill-mismatch in the U.S. economy. Finally, Section 6 concludes.

2 The Model

The model extends Arseneau and Chugh (2012) to include two-sided heterogeneity for workers and firms, skill-mismatch, and OTJ search. We follow the timing assumption in that model by using instantaneous hiring so that, while there are search frictions, any job match formed within a period becomes productive in that same period.

2.1 Households

The economy is inhabited by a unit mass of individuals, a fraction κ of which are low-skilled and the remaining fraction $1 - \kappa$ are high-skilled. Low-skilled individuals are only qualified for performing low-tech jobs, while high-skilled individuals can perform either high- or low-tech jobs. Mismatch occurs when a high-skill individual is matched with a low-tech job. In this case, there is an incentive to engage in OTJ search because the surplus arising from a high-skill individual matched with a high-tech job surpasses the surplus from mismatched employment.

Individuals are aggregated into two separate households, differentiated by type. For the sake of convenience, we assume there is aggregate risk sharing across individuals both within and between households.[6] Each household decides how much to consume, the number of state-contingent bonds to hold, and the mass of household members who participate in the labor force. Labor force participants are either employed or actively searching for jobs from a state of unemployment or mismatch employment. Employed individuals receive a wage while unemployed individuals receive a constant unemployment flow benefit. Individuals that are outside of the labor force enjoy the utility value of leisure.

[6] Assuming risk sharing within the household is common in search-theoretic general equilibrium models of the labor market following Merz (1995) and Andolfatto (1996). In this paper, we extend the risk sharing across households for analytic convenience. Additionally, this assumption helps focus directly on the mismatch distortion rather than interacting it with financial market imperfections.

2.1.1 Low-Skilled Households

The mass of low-skill individuals participating in the labor force is given by $lfp_t^L = n_t^L + (1 - f_t^L)s_t^L$, where: n_t^L denotes the mass of low-skill individuals working in low-tech jobs; s_t^L denotes the mass of low-skill individuals searching in the market for low-tech jobs; and f_t^L is the endogenous probability that a low-skill individual searching in the market for low-tech jobs matches with a low-tech firm (discussed below). Due to the timing of the model the mass of unemployed low-skill individuals at the end of the period is given by $u_t^L = (1 - f_t^L)s_t^L$ (successful search within a period is net out). Leisure obtained by a low-skill household is given by: $l_t^L = \kappa - lfp_t^L$.

The low-skilled household chooses sequences of consumption, c_t^L, state-contingent bond holdings, B_t^L, and search activity, s_t^L, to achieve a desired low-tech employment stock, n_t^L, in order to maximize discounted lifetime utility: $\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t (u(c_t^L) - h\left(lfp_t^L\right))$, where: $\mathbb{E}_t$ is the expectation operator; $\beta \in (0,1)$ is the exogenous subjective discount factor; u is utility from consumption, with $\frac{\partial u}{\partial c} = u_c > 0$ and $\frac{\partial^2 u}{\partial c^2} < 0$; and h is utility from leisure, with $\frac{\partial h}{\partial lfp} = h^{L\prime} > 0$ and $\frac{\partial^2 h}{\partial lfp^2} < 0$.[7]

Low-skilled households face the following budget constraint:

$$c_t^L + B_t^L = w_t^L n_t^L + \chi^L (1 - f_t^L)s_t^L + R_t B_{t-1}^L + \kappa\left(\Pi_t^L + \Pi_t^H\right),$$

where: w_t^L is the wage received by a low-skilled individual employed in a low-tech job; χ^L is an exogenous unemployment benefit paid to low-skill workers who searched for jobs but did not find one; the real state-contingent bond pays an interest rate of R_t; and Π_t^L and Π_t^H denote the profits of intermediate low- and high-tech goods producing firms (discussed below) paid to the household in the form of a dividend, which are taken as given when households solve their optimization problem. We assume that households receive a dividend from ownership in proportion to their share of the total population.

In addition to the budget constraint, the household also faces a constraint on the perceived law of motion for the stock of employment, n_t^L, given by:

$$n_t^L = (1 - \rho^L)n_{t-1}^L + f_t^L s_t^L,$$

which says that the number of low-skilled workers employed in low-tech jobs today is equal

[7] Matches between low-skill workers and high-tech jobs are not productive, so low-skill households will never choose to devote search activity to high-tech jobs. For expositional simplicity, we omit this choice.

the number employment relationships that existed in the previous period, net of those that terminate exogenously with probability ρ^L, plus new inflow. The new inflow is equal to the probability that a low-skill individual searching for a low-tech job matches with such job, f_t^L, multiplied by the number of searching individuals, s_t^L.

The matching probability, f_t^L, is equal to the ratio of matches to job seekers in the low-tech sector. Matches in the low-tech sector, $m_t^L = m^L(s_t^L + s_t^M, v_t^L)$, are increasing and concave in v_t^L, which denotes vacancies posted by low-tech firms (discussed below), and the total number of individuals searching for low-tech jobs. Total searchers in the low-tech sector are equal to the sum of unemployed low-skill searchers, s_t^L, and unemployed high-skill individuals searching for low-tech jobs, s_t^M (individuals who are mismatched are already matched with a low-tech job and therefore endogenously have no incentive to search for these jobs). Finally, define $\theta_t^L \equiv v_t^L/(s_t^L + s_t^M)$ as market tightness in the low-tech sector. High-skilled individuals engaged in search for mismatch employment can potentially crowd out low-skill search in the sense that $\partial\theta_t^L/\partial s_t^M < 0$.[8]

The first order conditions for c_t^L and B_t^L can be manipulated into a standard bond Euler equation:

$$1 = \mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^L}{u_{c,t}^L} R_{t+1} \right\}, \tag{1}$$

which defines the stochastic discount factor for pricing the one-period, risk-free government bond, $\Xi_{t+1|t} \equiv \beta u_{c,t+1}^L / u_{c,t}^L$.

We can also use the first order conditions on s_t^L and n_t^L to obtain the optimal labor-force participation condition for low-skilled individuals:

$$\frac{h_t^{L\prime}}{u_{c,t}^L} = (1 - f_t^L)\chi^L + f_t^L \left[w_t^L + (1-\rho^L)\mathbb{E}_t \Xi_{t+1|t} \left\{ \frac{1 - f_{t+1}^L}{f_{t+1}^L} \left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} - \chi^L \right) \right\} \right], \tag{2}$$

which says that the low-skilled household will search for low-tech employment up until the point at which the probability-weighted cost of doing so, the disutility of search effort net of the outside option, χ^L, is exactly offset by the probability weighted expected benefit of getting a low-tech job. The expected benefit of low-tech employment is the wage plus the

[8] Of course, it is also possible that high-skilled participation could crowd in low-skill search by increasing average and expected match quality in the low-tech sector, therefore inducing low-tech firms to post more vacancies. We address this possibility later.

continuation value of the long-lived employment relationship.

2.1.2 High-Skilled Households

The mass of high-skill individuals participating in the labor force is given by $lfp_t^H + lfp_t^M$, where: $lfp_t^H = n_t^H + (1 - f_t^H)s_t^H$; n_t^H denotes high-skill individuals working in high-tech jobs; s_t^H denotes high-skill individuals searching for high-tech jobs; and f_t^H is the probability a high-skill individual searching for a high-tech job matches with a high-tech firm (discussed below). Similarly, $lfp_t^M = n_t^M + (1 - f_t^L)s_t^M$, where: n_t^M denotes high-skill individuals working in low-tech jobs. The mass of unemployed high-skill individuals is u_t^H=$(1 - f_t^L)s_t^M + \left(1 - f_t^H\right) s_t^H$. Leisure obtained by a high-skill household is $l_t^H = 1 - \kappa - lfp_t^H - lfp_t^M$.

High-skilled households choose sequences of consumption, c_t^H, state-contingent bond holdings, B_t^H, and search activity in both the market for low- and high-tech jobs, given by s_t^M and s_t^H, respectively, in order to achieve a desired stock of mismatch and high-tech employment, given by n_t^M and n_t^H, respectively. Specifically, high-skilled household's utility maximization problem is:

$$\max \mathcal{U}^H = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(u(c_t^H) - h\left(lfp_t^H, lfp_t^M\right)\right)$$

subject to the budget constraint:

$$\begin{aligned} c_t^H + B_t^H = w_t^H n_t^H + w_t^M n_t^M + \chi^H \left[(1 - f_t^L)s_t^M + (1 - f_t^H)s_t^H\right] \\ +R_t B_{t-1}^H + (1 - \kappa)\left(\Pi_t^L + \Pi_t^H\right), \end{aligned}$$

and perceived laws of motion for the stocks of mismatch and high-tech employment:

$$n_t^M = \left(1 - \pi f_t^H\right)(1 - \rho^L)n_{t-1}^M + f_t^L s_t^M,$$

and

$$n_t^H = (1 - \rho^H)n_{t-1}^H + f_t^H s_t^H + \pi f_t^H (1 - \rho^L)n_{t-1}^M,$$

where: in the budget constraint w_t^H and w_t^M are the wages received by high-skilled individuals in high-tech and mismatch jobs, respectively; χ^H is an exogenous unemployment benefit paid to high-skill workers who searched for jobs but did not find one; and, in the laws of motion for high-tech and mismatch employment, $\pi \in (0,1)$ denotes the search efficiency of an OTJ searcher relative to that of an unemployed individual. Search on-the-job is as efficient as search from a state of unemployment when $\pi = 1$; in contrast, $\pi = 0$ shuts down OTJ search entirely.[9]

Any high-skilled individual engaged in OTJ search will accept a higher paying job in the high-tech sector, which occurs with probability πf_t^H. So, in terms of allocations, the first order effect of OTJ search is to increase the flow out of mismatch employment and into high-tech employment. This additional outflow is given by $\pi f_t^H(1-\rho^L)n_{t-1}^M$, which simply becomes an inflow into high-tech employment through job-to-job transition.

As with the market for low-tech jobs, the job finding probability f_t^H is equal to the ratio of matches to job seekers in the high-tech sector. Matches in the high-tech sector, $m_t^H = m^H(s_t^H + \pi(1-\rho^L)n_t^M, v_t^H)$, are increasing and concave in v_t^H, which denotes vacancies posted by high-tech firms, and the effective mass of individuals searching for high-tech jobs, $s_t^H + \pi(1-\rho^L)n_t^M$, which captures both high-skill unemployed individuals and OTJ searchers. Define $\theta_t^H \equiv v_t^H/(s_t^H + \pi(1-\rho^L)n_{t-1}^M)$ as market tightness in the market for high-tech jobs.

The first-order conditions over c_t^H and B_t^H can be combined to yield a standard consumption Euler equation:[10]

$$1 = \mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^H}{u_{c,t}^H} R_{t+1} \right\}. \tag{3}$$

Using this relationship in the first order condition for n_t^H, we can write the optimal participation condition in the market for high tech employment as:

$$\frac{h_t^{H\prime}}{u_{c,t}^H} = (1-f_t^H)\chi^H + f_t^H \left[w_t^H + (1-\rho^H)\mathbb{E}_t \Xi_{t+1|t} \left\{ \frac{1-f_{t+1}^H}{f_{t+1}^H} \left(\frac{h_{t+1}^{H\prime}}{u_{c,t+1}^H} - \chi^H \right) \right\} \right], \tag{4}$$

where: $h_t^{H\prime}$ is the derivative of the subutility of the high-skilled household over participation in the market for high-tech employment ($h^{H\prime} > 0$ and $h^{H\prime\prime} > 0$). This equation has a similar interpretation to equation (2) above.

Finally, the condition governing optimal participation for high-skilled individuals in the market for low-tech jobs can be written as:

[9] Mismatched individuals endogenously have no incentive to direct OTJ search to the low-tech sector, so we omit this choice margin for simplicity.

[10] Note that complete financial markets implies $\beta u_{c,t+1}^L/u_{c,t}^L = \beta u_{c,t+1}^H/u_{c,t}^H = \Xi_{t+1|t}$.

$$\frac{h_t^{M\prime}}{u_{c,t}^H} = \left(1 - f_t^L\right)\chi^H \tag{5}$$

$$+f_t^L\left[w_t^M + (1-\rho^L)\mathbb{E}_t\Xi_{t+1|t}\left\{\left[\frac{1-f_{t+1}^L}{f_{t+1}^L} - \pi\left(1 - \frac{f_{t+1}^H}{f_{t+1}^L}\right)\right]\left(\frac{h_{t+1}^{M\prime}}{u_{c,t+1}^H} - \chi^H\right)\right\}\right],$$

where: $h_t^{M\prime}$ is the derivative of the subutility of the high-skilled household over participation in the market for mismatch employment ($h^{M\prime} > 0$ and $h^{M\prime\prime} > 0$). When OTJ search is shut down, so that $\pi = 0$, the interpretation of this equation is identical to that of equations (2) and (4). The only difference for $\pi > 0$ is that the continuation value of a mismatch job is adjusted for the possibility that successful OTJ search shortens the expected duration of a mismatch employment relationship. The size of this adjustment is increasing in the relative ease with which a match can be made in the high-tech sector, f_{t+1}^H/f_{t+1}^L.

The household's first order conditions must hold regardless of the particular value of the OTJ search efficiency parameter π. In order for the household to have the same flexibility over its choice of OTJ search activity as it does over its search unemployment margins (the extent of search unemployment is determined by the choices of s_t^H and s_t^M, whose values are not barred from being zero), we assume that it faces the optimization subproblem:

$$\max_{\pi^* \in \{0,\pi\}}\left\{\max_{c_t^H, B_t^H, s_t^M, s_t^H, n_t^M, n_t^H} \mathcal{U}^H\right\}.$$

Therefore, the household's choices n_t^M and π^* are such that they implicitly and jointly determine the extent of OTJ search activity. Indeed, letting s_t^{OTJ} denote OTJ search activity, then $s_t^{OTJ} \equiv (1-\rho^L)\pi n_t^M$.[11]

The tradeoff the high-skilled household faces when deciding to engage in search for mismatch employment can be highlighted by imposing steady state on equations (4) and (5). For simplicity, assume no unemployment benefits, so $\chi^H = 0$, and also assume that while employment relationships form in the presence of search frictions they only last for a single

[11] The choice margin over π allows for the possibility that mismatch employment occurs in the private equilibrium ($s^M > 0$) but OTJ search does not ($\pi = 0$). As shown below, our baseline calibration implies both $s_M > 0$ and $\pi^* = \pi > 0$ so that the high-skill household optimally chooses both to participate in the low-tech market and to engage in OTJ search activity. These decisions offer additional validity to our calibration and theoretical framework because absent the choice of $\pi^* > 0$ the baseline economy would be inconsistent with data. Indeed, many papers have documented the extent to which OTJ search and job-to-job transitions are an important empirical feature of the labor market.

period so that $\rho^L = \rho^H = 1$. In this special case, the high-skilled household would allocate search activity so that the marginal rate of substitution between participation in the high-tech and mismatch labor markets is equal to the probability adjusted wage ratio across the two markets:

$$\frac{h^{H\prime}}{h^{M\prime}} = \frac{f^H}{f^L}\frac{w^H}{w^M}.$$

Intuitively, holding $h^{H\prime}/h^{M\prime}$ constant, a larger wage premium for working in high-tech employment (higher w^H/w^M) must be compensated by improved job finding prospects in the market for mismatch employment (offset by lower f^H/f^L). In this sense, our model captures the idea that high-skilled individuals are willing to accept a lower quality job in order to move out of unemployment more quickly, but doing so comes at the cost of having to accept a lower wage in a job for which they are over-qualified.

2.2 Production

The production side of the economy is divided into a final goods sector and an intermediate goods sector.

2.2.1 Final Goods Production

The representative final goods producer purchases both low- and high-tech intermediate inputs, denoted y_t^L and y_t^H, respectively. It aggregates these intermediate goods into a final good using the technology $Z_t F(y_t^L, y_t^H)$, where: Z_t is total factor productivity; and F is increasing and concave in each of its arguments. This final good is then sold to households in a perfectly competitive market for the final consumption good. The final goods producer chooses intermediate inputs to solve the following problem:

$$\max \ \mathbb{E}_t \sum_{t=0}^{\infty} \Xi_{t+1|t} \left[Z_t F(y_t^L, y_t^H) - p_t^L y_t^L - p_t^H y_t^H \right],$$

where: p_t^L and p_t^H are the prices of the low- and high-tech intermediate inputs, respectively, relative to the final good. The demand for each intermediate input equates the marginal product to the price, so that $Z_t F_{L,t} = p_t^L$ and $Z_t F_{H,t} = p_t^H$.

2.2.2 Intermediate Goods Production

Both low- and high-tech firms use labor to produce their respective intermediate input, which is then sold to the final goods producer. Regardless of firm type, the intermediate goods producer engages in costly search and matching in order to find a worker before production can take place. The low-tech intermediate goods producer pays a fixed exogenous flow cost, γ^L, to post a vacancy for an open position in the low-tech market. Similarly, the high-tech intermediate goods producer pays a fixed exogenous flow cost, γ^H, to post a vacancy for an open position in the high-tech market. We assume free entry into vacancy posting.

Low-tech Firms For a given low-tech vacancy, the low-tech firm can hire either a low- or a high-skilled worker. Both labor inputs are used to produce the low-tech intermediate good according to the production technology $y_t^L = g^L(n_t^L, n_t^M)$, where: g^L is increasing and concave in each of its arguments.

The low-tech firm chooses the desired stock of low-skill employees, n_t^L, the desired stock of high-skill employees, n_t^M, and vacancies, v_t^L, to solve:

$$\max \ \mathbb{E}_t \sum_t \Xi_{t+1|t} \left[p_t^L g^L(n_t^L, n_t^M) - w_t^L n_t^L - w_t^M n_t^M - \gamma^L v_t^L \right],$$

subject to the firm's perceived laws of motion for low-skill and mismatch employment stocks, respectively:

$$n_t^L = (1-\rho^L) n_{t-1}^L + \eta_t^L q_t^L v_t^L,$$

and

$$n_t^M = \left(1 - \pi f_t^H\right)(1-\rho^L) n_{t-1}^M + \left(1 - \eta_t^L\right) q_t^L v_t^L,$$

where: q_t^L is the probability that a given vacancy posted in the market for low-tech jobs is successful in finding a worker, regardless of whether the worker is low- or high-skill. Furthermore, the fraction of low-skill workers in the total pool of individuals searching for low-skill jobs is given by $\eta_t^L \equiv s_t^L/(s_t^L + s_t^M)$. With this notation, the per period probability that a low-tech vacancy turns into an employment match with a low-skill worker is $\eta_t q_t^L$ and the probability that a low-tech vacancy turns into a mismatch employment relationship with a high-skill worker is $(1 - \eta_t) q_t^L$. Also, note that the low-tech firm will lose high-skill workers who are successful in OTJ search with probability πf_t^H.

Total employment in the low-tech sector is given by $N_t^L = n_t^L + n_t^M$ so that the sectoral mismatch rate is given by n_t^M/N_t^L. In addition, total aggregate employment is $N_t = n_t^L + n_t^M + n_t^H$. Thus, the aggregate mismatch rate is n_t^M/N_t. Furthermore, the average wage in the low-tech sector is $W_t^L = (w_t^L n_t^L + w_t^M n_t^M)/N_t^L$.

The first order condition on v_t^L gives:

$$\frac{\gamma^L}{q_t^L} = \eta_t^L \mathbf{J_t^L} + \left(1 - \eta_t^L\right) \mathbf{J_t^M}, \tag{6}$$

where: $\mathbf{J_t^L}$ and $\mathbf{J_t^M}$ are defined by the Lagrangian multipliers on the perceived laws of motion for low-tech and mismatch employment, respectively. The low-tech firm posts vacancies up until the point at which the cost, γ^L, is exactly offset by the expected gain from making a match. The expected gain is the probability that a match is made in the low-tech market, q_t^L, times a probability weighted average of the value of a match with a low-tech worker, $\eta_t^L \mathbf{J_t^L}$, and a high-tech worker, $(1 - \eta_t^L)\mathbf{J_t^M}$.

The first-order conditions for n_t^L and n_t^M give expressions for the value to the firm of both types of matches:

$$\mathbf{J_t^L} = p_t^L g_{n_t^L}^L - w_t^L + \left(1 - \rho^L\right) \mathbb{E}_t \left\{\Xi_{t+1|t} \mathbf{J_{t+1}^L}\right\}, \tag{7}$$

and

$$\mathbf{J_t^M} = p_t^L g_{n_t^M}^L - w_t^M + \left(1 - \rho^L\right) \mathbb{E}_t \left\{\Xi_{t+1|t} \left(1 - \pi f_{t+1}^H\right) \mathbf{J_{t+1}^M}\right\}. \tag{8}$$

Equation (7) equates the value of a low-skilled employee working in the low-tech job to the present discounted value of the stream of marginal revenue net of the wage over the expected duration of the employment relationship. Equation (8) is interpreted in a similar way with the exception that OTJ search effectively lowers the continuation value of mismatch employment owing to the outflow into high-tech employment stemming from on-the-job search.

High-tech Firms High-tech firms only employ high-skilled workers because those workers are the only ones qualified to do the job. Production is given by the following: $y_t^H = g^H(n_t^H)$, where: g^H is increasing and concave. The high-tech firm chooses the stock of high-skill employees and vacancies to solves the following profit maximization problem:

$$\max \ \mathbb{E}_t \sum_t \Xi_{t+1|t} \left[p_t^H g^H(n_t^H) - w_t^H n_t^H - \gamma^H v_t^H\right],$$

subject to the perceived law of motion for high-tech employment:

$$n_t^H = (1 - \rho^H) n_{t-1}^H + q_t^H v_t^H,$$

where: q_t^H is the probability that a given vacancy posted in the market for high-tech jobs is successful in finding a worker. The average wage in the high-tech sector is simply w_t^H, whereas the average wage of high-skill individuals is $W_t^H = (w_t^H n_t^H + w_t^M n_t^M)/(n_t^H + n_t^M)$.

The first order condition on v_t^H gives:

$$\gamma^H/q_t^H = \mathbf{J_t^H}, \tag{9}$$

where: $\mathbf{J_t^H}$ is the Lagrangian multiplier on the perceived law of motion for high-tech employment.

The high-tech firm's first order conditions for n_t^H gives the following expression for the firm's value of a match:

$$\mathbf{J_t^H} = p_t^H g_{n_t^H}^H - w_t^H + \left(1 - \rho^H\right) \mathbb{E}_t \left\{ \Xi_{t+1|t} \mathbf{J_{t+1}^H} \right\}. \tag{10}$$

We could express equations (9) and (10) as a single condition so that $\gamma^H/q_t^H = p_t^H g_{n_t^H}^H - w_t^H + (1 - \rho^H)\mathbb{E}_t\{\Xi_{t+1|t}(\gamma^H/q_{t+1}^H)\}$.

2.3 The Labor Market

In order to close the model, we need to address matching and wage determination in each of the two labor markets.

2.3.1 Matching

Labor market matches are formed according to a constant returns matching technology in both the market for low- and high-tech jobs. Aggregate employment of low-skilled workers employed in low-tech jobs evolves according to:

$$n_t^L = (1 - \rho^L) n_{t-1}^L + \eta_t^L m_t^L. \tag{11}$$

As noted above, η_t^L is the probability that a match in the market for low-tech employment is formed with a low-skill worker, so that, recall, $\eta_t^L \equiv s_t^L/(s_t^L + s_t^H)$ is endogenously determined by the search activity of low- and high-skilled individuals.

The law of motion for mismatch employment is given by:

$$n_t^M = (1 - \rho^L) n_{t-1}^M - \eta_t^H m_t^H + (1 - \eta_t^L) m_t^L, \tag{12}$$

where: $\eta_t^H \equiv \pi(1 - \rho^L) n_{t-1}^M/(s_t^H + \pi(1 - \rho^L) n_{t-1}^M)$ is the probability that a given match made in the high-tech labor market is made with an OTJ searcher.

Finally, the law of motion for high-tech jobs is:

$$n_t^H = (1-\rho^H)n_{t-1}^H + m_t^H. \tag{13}$$

2.3.2 Wage Determination

Wages are determined through Nash bargaining subject to instantaneous renegotiation and without commitment to the future path of wages.[12] We use Nash bargaining for no other reason than it is analytically convenient; it allows us to anchor our analysis within the standard Hosios condition for efficient surplus splitting (Hosios, 1990) in a straightforward way.[13]

Let $\psi^i \in (0,1)$ for $i \in \{L,H\}$ denote the exogenous bargaining power of workers. For the sake of brevity we present only the wages that solve the bargaining problem, leaving the details—including a full derivation of the fundamental value functions used in the bargaining problem itself—to the Appendix.

The wage for a low-skilled worker employed in a low-tech job is given by:

$$w_t^L = \psi^L p_t^L g_{n_t^L}^L + \left(1-\psi^L\right)\chi^L + \psi^L(1-\rho^L)\mathbb{E}_t\left\{\Xi_{t+1|t} f_{t+1}^L \mathbf{J_{t+1}^L}\right\}. \tag{14}$$

Therefore, the wage paid by low-tech firms to low-skilled workers is a weighted average of the present discounted value of the stream of marginal revenue that accrues to the low-tech firm from hiring the additional employee and the outside option that accrues to the worker, given by the unemployment benefit.

The mismatch wage is given by the expression:

$$\begin{aligned} w_t^M &= \psi^H p_t^L g_{n_t^M}^L + \left(1-\psi^H\right)\chi^H \\ &+\psi^H(1-\rho^L)\mathbb{E}_t\left\{\Xi_{t+1|t}\left[\left(1-\pi f_{t+1}^H\right) f_{t+1}^L \mathbf{J_{t+1}^M} - \pi\left(1-f_{t+1}^H\right) f_{t+1}^H \mathbf{J_{t+1}^H}\right]\right\}. \end{aligned} \tag{15}$$

All else equal, the possibility of OTJ search depresses the mismatch wage through two separate channels. First, securing a mismatch job and subsequently engaging in OTJ search

[12] This assumption implies that, regardless of employment status, the workers' threatpoint in wage negotiations is always the value of unemployment. As a result it allows us to circumvent the issues raised in Shimer (2006) regarding the use of Nash bargaining in models with OTJ search.

[13] To be precise, the assumption of Nash bargaining is irrelevant for the main points we want to make regarding efficiency in Section 4. When we evaluate the mismatch distortion, we do so under the assumption that the match surplus is split efficiently There are any number of alternative ways to implement efficient surplus splits—including, for example, wage posting in a competitive search equilibrium—suggesting there is nothing special about using Nash bargaining in driving our main qualitative results.

opens another avenue through which a high-skilled worker can eventually move into high-tech employment. The high-skilled worker is willing to accept a lower wage to have access to this opportunity. Second, the firm needs to be compensated for the fact that OTJ search results in mismatch employment relationships have a shorter expected duration. This is captured by the term $(1 - \pi f^H_{t+1}) f^L_{t+1} \mathbf{J^M_{t+1}}$. In the absence of OTJ search ($\pi = 0$), the continuation value reduces to $\psi^H(1 - \rho^L)\mathbb{E}_t\{\Xi_{t+1|t} f^L_{t+1} \mathbf{J^M_{t+1}}\}$ and the mismatch wage takes a similar form as equation (14).

Finally, the wage for a high-skilled worker employed in a high-tech job is given by:

$$w^H_t = \psi^H p^H_t g^H_{n^H_t} + \left(1 - \psi^H\right)\chi^H + \psi^H(1 - \rho^H)\mathbb{E}_t\left[\Xi_{t+1|t}\left(f^H_{t+1}\mathbf{J^H_{t+1}}\right)\right]. \tag{16}$$

Note that because free entry into vacancy postings drives $\mathbf{J^H_{t+1}} = \gamma^H/q^H_t$, the continuation value can also be expressed as $\mathbb{E}_t\{\Xi_{t+1|t}(f^H_{t+1}/q^H_t)\gamma^H]$.

2.4 Search Equilibrium

Given the exogenous process for technology, Z_t, the equilibrium of the system is a sequence of allocations and prices $\{c^L_t, c^H_t, R_t, n^L_t, n^M_t, n^H_t, s^L_t, s^M_t, s^H_t, v^L_t, v^H_t, \mathbf{J^L_t}, \mathbf{J^M_t}, \mathbf{J^H_t}, w^L_t, w^M_t, w^H_t, p^L_t, p^H_t\}$ that solves the optimality conditions for: low-skilled households, summarized by equations (1) through (2); high-skilled households, summarized be equations (3) through (5); demand for the low- and high-tech intermediate input, given by $F_{L,t} = p^L_t/Z_t$ and $F_{H,t} = p^H_t/Z_t$, respectively; low-tech intermediate goods producers, summarized by equations (6) through (8); high-tech intermediate goods producers, summarized by equations (9) and (10). We also have the laws of motion for respective employment stocks, equations (11) through (13); and the wage expressions, equations (14) through (16).

In addition, we have the economy-wide resource constraint:

$$Y_t = c^L_t + c^H_t + \gamma^L v^L_t + \gamma^H v^H_t. \tag{17}$$

3 Social Efficiency

We define social efficiency as an equally-weighted sum of the utility of low- and high-skilled households whose discounted lifetime expected value is denoted by U. With this definition, the efficient allocations $\{c^L_t, c^H_t, n^L_t, n^M_t, n^H_t, s^L_t, s^M_t, s^H_t, v^H_t, v^L_t, \eta^L_t, \eta^H_t\}$ are characterized by a set of 12 equations that include: equalization of the marginal rate of consumption for low- and high-skilled individuals, a set of two static labor market efficiency conditions; a set of three dynamic labor market efficiency conditions; the economy-wide resource constraint; a set of three laws of motion for the respective employment stocks; and, finally, two equations defining η_t^L and η_t^H, respectively. Akin to the decentralized high-skill household's utility maximization problem, we assume that the planning problem involves a utility optimization subproblem that selects $\pi^* \in \{0, \pi\}$. Details for the solution to the social planner's problem are provided in the Appendix. For the sake of brevity, in what follows we concentrate only on the set of static and dynamic efficiency conditions that summarize the labor market.

The static efficiency condition for overall search activity directed toward the market for low-tech employment is given by:

$$\frac{m_{s,t}^L}{m_{v,t}^L}\gamma^L = \eta_t^L \frac{h_t^{L\prime}}{u_{c,t}^L} + \left(1 - \eta_t^L\right) \frac{h_t^{M\prime}}{u_{c,t}^H}, \tag{18}$$

where: $m_{s,t}^L$ and $m_{v,t}^L$ denote the derivative of the low-tech matching function with respect to total search activity in that market and with respect to vacancies in that market, respectively. This expression equates the static marginal rate of transformation (MRT) of a unit of leisure into a unit of the final consumption good through the low-tech intermediate input (on the right left side) to a weighted average of the static marginal rates of substitution (MRS) between consumption and leisure for low- and high-skilled individuals (on the right hand side).[14]

Intuitively, within the period there are two distinct ways for the social planner to transform a unit of leisure into the final consumption good through the production of the low-tech intermediate good. The first is through the participation of low-skilled individuals, where the effectiveness of a unit of search in the matching pool for low-tech jobs is governed by the probability η_t^L. This unit of search is transformed into productive labor, which is then ultimately used in production, through the matching function (captured by the left hand side of equation (18)). Alternatively, the planner can achieve the same outcome through high-skilled individuals, transforming an effective unit of search into mismatch employment

[14] See Arseneau and Chugh (2012) for a more detailed description of how to interpret both the static and dynamic efficiency conditions in a general equilibrium labor search model and, in particular, how to think about the marginal rate of transformation in this class of models.

with probability $1-\eta_t^L$. The effectiveness of a unit of mismatch search in the matching pool for low-tech jobs is governed by the probability $1-\eta_t^L$. This probability links the MRS between consumption and leisure for low- and high-skilled (mismatched) individuals in the socially efficient equilibrium.

For high-tech employment, the static efficiency condition is:

$$\gamma^H \frac{m_{s,t}^H}{m_{v,t}^H} + \eta_t^H f_t^H \frac{\gamma^L}{m_{v,t}^L} = \left(1-\eta_t^H f_t^H\right) \frac{h_t^{H\prime}}{u_{c,t}^H} + \eta_t^H f_t^H \left(\frac{\eta_t^L}{f_t^L} \frac{h_t^{L\prime}}{u_{c,t}^L} + \left(1-\frac{\eta_t^L}{f_t^L}\right) \frac{h_t^{M\prime}}{u_{c,t}^H}\right), \tag{19}$$

where: $m_{s,t}^H$ and $m_{v,t}^H$ denote the derivative of the high-tech matching function with respect to total search activity in that market and with respect to vacancies in that market, respectively. The interpretation of equation (19) is broadly similar to that of equation (18) but is complicated by the role of OTJ search. In absence of OTJ search, so that $\eta_t^H = 0$, the expression simplifies to $h_t^{H\prime}/u_{c,t}^H = \gamma^H m_{s,t}^H / m_{v,t}^H$, which equates the MRS between consumption and participation in the market for high-tech jobs to the MRT of a unit of leisure into a unit of the final consumption good through the high-tech intermediate input. The opportunity for OTJ search through mismatch employment ($\eta_t^H > 0$) opens up another channel through which the planner can transform a unit of leisure of the high-skilled individual into a the high-tech intermediate output.

In addition to the two static conditions, the efficient equilibrium is also characterized by a set of three dynamic efficiency conditions for low- and high-tech job creation and for the creation of mismatch jobs. The dynamic efficiency condition for the creation of low-tech jobs staffed by low-skill workers is given by:

$$\frac{\gamma^L}{m_{v,t}^L} = Y_{1,t} + \left(1-\eta_t^L\right)\Gamma_t^L + \left(1-\rho^L\right)\mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^L}{u_{c,t}^L} \left(\frac{\gamma^L}{m_{v,t+1}^L} - \left(1-\eta_{t+1}^L\right)\Gamma_{t+1}^L - \frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} \right) \right\}, \tag{20}$$

where: $Y_{1,t}$ is the derivative of the final goods (aggregate) production function with respect to low-skilled labor; and we define $\Gamma_t^L \equiv \frac{1}{f_t^L}\left(\frac{h_t^{L\prime}}{u_{c,t}^L} - \frac{h_t^{M\prime}}{u_{c,t}^H}\right) = \frac{1}{\eta_t^L}\frac{1}{f_t^L}\left(\frac{h_t^{M\prime}}{u_{c,t}^H} - \frac{m_{s,t}^L}{m_{v,t}^L}\gamma^L\right)$, where the second equality comes from the static efficiency condition, equation (18). The dynamic efficiency condition above ensures that the social cost of generating a low-tech job staffed by a low-skilled worker is exactly offset by the discounted expected benefit. The left hand side is the cost of posting the low-tech vacancy (normalized by the number of new matches generated by an additional vacancy posting). The right hand side is the expected marginal social gain, which consists of three separate terms: (1.) the marginal product of low-skilled labor; (2.) net of the potential benefit (cost) owing to being able to staff the job with a low-skill individual that has a higher (lower) MRS between consumption and leisure, $\Gamma_t^L > (<) \ 0$; and (3.) the continuation value of a low-tech match.

The dynamic efficiency condition for the creation of mismatch jobs is given by:

$$\frac{\gamma^L}{m_{v,t}^L} = Y_{2,t} - \eta_t^L \Gamma_t^L + \left(1-\rho^L\right) \mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^L}{u_{c,t}^L} \left(\begin{array}{c} (1-\pi f_{t+1}^H)\left(\frac{\gamma^L}{m_{v,t+1}^L} + \eta_{t+1}^L \Gamma_{t+1}^L - \frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L}\right) \\ +\pi(1-f_{t+1}^H)\frac{h_{t+1}^{H\prime}}{u_{c,t+1}^H} \end{array} \right) \right\}, \tag{21}$$

where: $Y_{2,t}$ is the derivative of the final goods (aggregate) production function with respect to mismatched labor. The interpretation here is similar to that of equation (20) above with two exceptions. First, the sign on the Γ_t^L term is different. This means that for any parameterization in which $\Gamma_t^L > (<) \ 0$, mismatch employment increases (decreases) the marginal benefit of a low-skilled worker and decreases (increases) the marginal benefit of a mismatched worker. In other words, what is beneficial to one market is necessarily costly to the other. Second, OTJ search affects the continuation value of a mismatch job. Intuitively, the more likely a high-skilled individual is to find a high-tech job while employed by the low-tech firm, the lower will be the continuation value simply due to the shorter expected duration of the mismatch job.

Finally, the dynamic efficiency condition for the creation of high-tech jobs is given by:

$$\frac{\gamma^H}{m_{v,t}^H} = Y_{3,t} - \Gamma_t^H + \left(1-\rho^H\right) \mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^H}{u_{c,t}^H} \left(\frac{\gamma^H}{m_{v,t+1}^H} + \Gamma_{t+1}^H - \frac{h_{t+1}^{H\prime}}{u_{c,t+1}^H} \right) \right\}, \tag{22}$$

where: $Y_{3,t}$ is the derivative of the final goods (aggregate) production function with respect to high-skilled labor; and we define $\Gamma_t^H \equiv \frac{1}{f_t^H}\left(\frac{h_t^{H\prime}}{u_{c,t}^H} - \frac{m_{s,t}^H}{m_{v,t}^H}\gamma^H\right) = \eta_t^H \frac{\gamma^L}{m_{v,t}^L} + \eta_t^H \frac{\eta_t^L}{f_t^L}\left(\frac{h_t^{M\prime}}{u_{c,t}^H} - \frac{h_t^{L\prime}}{u_{c,t}^L}\right) + \eta_t^H \left(\frac{h_t^{H\prime}}{u_{c,t}^H} - \frac{h_t^{M\prime}}{u_{c,t}^H}\right)$. Notice that shutting down OTJ search, so that $\eta_t^H = 0$ implying $h_t^{H\prime}/u_{c,t}^H = \left(m_{s,t}^H/m_{v,t}^H\right)\gamma^H$, means that both the static and the dynamic social efficiency conditions for the high-tech market are largely independent of developments in the low-tech market.

4 Characterizing the Distortion

We demonstrate the conditions under which the competitive search equilibrium coincides with the socially efficient equilibrium. Our approach is to manipulate the description of the private competitive search equilibrium into a set of conditions that take a similar form as what was presented in the previous section on social efficiency. To the degree that the private and socially optimal efficiency conditions do not perfectly coincide, we use the difference between the two to define a wedge that summarizes the distortion associated with a particular margin. We assume throughout that the matching functions are Cobb-Douglas with the elasticity of matches with respect to total search activity in the low- and high-tech markets denoted ξ^L and ξ^H, respectively. Our focus is on the distortionary effects of labor market mismatch and OTJ search. Details of all derivations are given in the Appendix.

4.1 Static Distortions

The static labor efficiency condition for low-tech jobs in the private equilibrium is derived using the low-tech firm's optimal vacancy posting condition, equation (6), the low-skill household's optimal search condition, given by equation (2), the Nash sharing rule, and the high-skill household's optimal search condition for mismatch employment, equation (5). The resulting expression is solved for the quantity $\gamma^L \left(m^L_{s,t}/m^L_{v,t} \right)$. This is compared to the corresponding static socially efficient condition for low-tech job creation, equation (18). The left hand sides are equal, but the right hand sides are potentially different.

Accordingly, we can define the low-tech static wedge, $\Omega^L_{Static,t}$, by taking a ratio of the two expressions resulting in the following:

$$\Omega^L_{Static,t} \equiv \frac{\eta^L_t \frac{h^{L\prime}_t}{u^L_{c,t}} + \left(1-\eta^L_t\right) \frac{h^{M\prime}_t}{u^H_{c,t}}}{\frac{1-\psi^L}{\psi^L}\frac{\xi^L}{1-\xi^L}\left[\eta^L_t\left(\frac{h^{L\prime}_t}{u^L_{c,t}} - \chi^L\right) + \left(1-\eta^L_t\right)\left(\frac{h^{M\prime}_t}{u^H_{c,t}} - \chi^H\right)\frac{\psi^L\left(1-\psi^H\right)}{\left(1-\psi^L\right)\psi^H}\right]}. \tag{23}$$

We follow a similar strategy to get an expression for the static labor efficiency condition for high-tech jobs in the private equilibrium. As above, we use the high-tech firm's optimal vacancy posting condition, equation (9), the high-skill household's optimal search condition for high-tech jobs, equation (4), and the Nash sharing rule to isolate $\gamma^H \left(m^H_{s,t}/m^H_{v,t} \right)$. The resulting expression is then compared to the socially efficient condition, equation (19), in order to define the static wedge for high-tech employment, $\Omega^H_{Static,t}$:

$$\Omega^H_{Static,t} \equiv \frac{\left(1-\eta^H_t f^H_t\right)\frac{h^{H\prime}_t}{u^H_{c,t}} + \eta^H_t f^H_t \left(\frac{\eta^L_t}{f^L_t}\frac{h^{L\prime}_t}{u^L_{c,t}} + \left(1-\frac{\eta^L_t}{f^L_t}\right)\frac{h^{M\prime}_t}{u^H_{c,t}} - \frac{\gamma^L}{m^L_{v,t}}\right)}{\frac{1-\psi^H}{\psi^H}\frac{\xi^H}{1-\xi^H}\left(\frac{h^{H\prime}_t}{u^H_{c,t}} - \chi^H\right)}. \tag{24}$$

4.2 Dynamic Distortions

We derive the dynamic distortion for low-tech jobs in three steps. First, we substitute the corresponding Nash wage into the low-tech firm's job creation condition for jobs staffed by low-skilled workers and, where necessary, apply the optimal low-tech vacancy posting condition. The resulting expression is solved for the quantity $\gamma^L/m^L_{v,t+1}$. In the second step, we do the same thing for the low-tech firm's job creation condition for jobs staffed by mismatched high-skilled workers and solve the resulting expression for $\gamma^L/m^L_{v,t+1}$. Finally, we multiply right and left hand sides of the former equation by η^L_t and the later equation by $1-\eta^L_t$ and add the two together. After some additional algebra, the resulting expression can then be expressed as a ratio to equation (20) from the social planning problem. We define the resulting expression as the dynamic distortion for low-tech job creation:

$$\Omega^L_{Dynamic,t} \equiv \frac{Y_{1,t} + \left(1-\eta^L_t\right)\Gamma^L_t + \left(1-\rho^L\right)\mathbb{E}_t\Xi_{t+1}\left[\frac{\gamma^L}{m^L_{v,t+1}} - \left(1-\eta^L_{t+1}\right)\Gamma^L_{t+1} - \frac{h^{L\prime}_{t+1}}{u^L_{c,t+1}}\right]}{\left(\begin{array}{c}\frac{1-\psi^L}{1-\xi^L}\eta^L_t\left(Y_{1,t}-\chi^L\right) + \frac{1-\psi^H}{1-\xi^L}\left(1-\eta^L_t\right)\left(Y_{2,t}-\chi^H\right)\\ +(1-\rho^L)\mathbb{E}_t\Xi_{t+1|t}\left\{\eta^L_t\Lambda^L_{t+1} + \left(1-\eta^L_t\right)\Lambda^M_{t+1}\right\}\\ +\pi\left(1-\eta^L_t\right)\mathbb{E}_t\Xi_{t+1|t}\left\{(1-\rho^H)\Lambda^H_{t+1} - (1-\rho^L)\frac{m^H_{s,t+1}}{\xi^H}\Lambda^M_{t+1}\right\}\end{array}\right)}, \tag{25}$$

where: Γ^L_t is defined as in Section 3 and we define $\Lambda^L_t \equiv \left(1-\frac{\psi^L}{\xi^L}m^L_{s,t}\right)\frac{1-\psi^L}{1-\xi^L}\frac{\xi^L}{\psi^L}\frac{1}{m^L_{s,t}}\left(\frac{h^{L\prime}_t}{u^L_{c,t}} - \chi^L\right)$; $\Lambda^M_t \equiv \left(1-\frac{\psi^H}{\xi^L}m^L_{s,t}\right)\frac{1-\psi^H}{1-\xi^L}\frac{\xi^L}{\psi^H}\frac{1}{m^L_{s,t}}\left(\frac{h^{M\prime}_t}{u^H_{c,t}} - \chi^H\right)$; and $\Lambda^H_t \equiv \left(1-\frac{1}{\xi^H}m^H_{s,t}\right)\frac{1-\psi^H}{1-\xi^L}\left(\frac{h^{H\prime}_t}{u^H_{c,t}} - \chi^H\right)$. The dynamic wedge for low-tech employment measures the gap between the discounted expected return to investing in low-tech job creation in the private versus socially efficient equilibrium.

A similar derivation gives rise to the dynamic distortion for mismatch job creation:

$$\Omega^M_{Dynamic,t} \equiv \frac{Y_{2,t} - \eta^L_t \Gamma^L_t + \left(1-\rho^L\right) \mathbb{E}_t \Xi_{t+1} \left[\begin{array}{c} (1-\pi f^H_{t+1}) \left(\frac{\gamma^L}{m^L_{v,t+1}} + \eta^L_{t+1}\Gamma^L_{t+1} - \frac{h^{L\prime}_{t+1}}{u^L_{c,t+1}} \right) \\ +\pi(1-f^H_{t+1}) \frac{h^{H\prime}_{t+1}}{u^H_{c,t+1}} \end{array} \right]}{\left(\begin{array}{c} \frac{1-\psi^L}{1-\xi^L} \eta^L_t \left(Y_{1,t} - \chi^L\right) + \frac{1-\psi^H}{1-\xi^L} \left(1-\eta^L_t\right) \left(Y_{2,t} - \chi^H\right) \\ +(1-\rho^L) \mathbb{E}_t \Xi_{t+1|t} \left\{ \eta^L_t \Lambda^L_{t+1} + \left(1-\eta^L_t\right) \Lambda^M_{t+1} \right\} \\ +\pi \left(1-\eta^L_t\right) \mathbb{E}_t \Xi_{t+1|t} \left\{ (1-\rho^H)\Lambda^H_{t+1} - (1-\rho^L) \frac{m^H_{s,t+1}}{\xi^H} \Lambda^M_{t+1} \right\} \end{array} \right)}. \tag{26}$$

In comparing equations (25) and (26), one thing that stands out is that the denominators (which describe the allocations in the private economy) are identical. In contrast, in the presence of mismatch employment ($\eta^L_t < 1$), the numerators (which describe the allocations in the socially efficient equilibrium) need not be equal. This is true regardless of whether or not there is OTJ search.

Finally, we derive the dynamic distortion for high-tech jobs by first substituting the Nash wage for high-skilled workers into the high-tech firm's job creation condition. The resulting expression is written as a ratio to equation (22) for the social planning problem allowing us to define an expression for the dynamic distortion for high-tech job creation:

$$\Omega^H_{Dynamic,t} \equiv \frac{Y_{3,t} - \Gamma^H_t + \left(1-\rho^H\right) \mathbb{E}_t \Xi_{t+1} \left[\frac{\gamma^H}{m^H_{v,t+1}} + \Gamma^H_{t+1} - \frac{h^{H\prime}_{t+1}}{u^H_{c,t+1}} \right]}{\frac{1-\psi^H}{1-\xi^H} \left(Y_{3,t} - \chi^H\right) + \left(1-\rho^H\right) \mathbb{E}_t \Xi_{t+1|t} \left[\frac{\gamma^H}{m^H_{v,t+1}} \left(1 - \frac{\psi^H}{\xi^H} m^H_{s,t+1}\right) \right]}. \tag{27}$$

As above, the dynamic wedge for high-tech employment measures the gap between the discounted expected return to investing in high-tech job creation in the private versus socially efficient equilibrium.

4.3 Four Special Cases

We present four special cases which, taken together, provide a complete characterization of the effect of mismatch and on-the-job search—both separately and together—on the standard search-based labor wedges derived in Arseneau and Chugh (2012).

Unless otherwise noted, the Hosios condition is assumed to hold in the markets for both low- and high-tech jobs, so that $\xi^L = \psi^L$ and $\xi^H = \psi^H$, and there are no unemployment benefits, $\chi^L = \chi^H = 0$. We make these assumption because it zeros out both the congestion externality generated by inefficient surplus splits and the distortion created by unemployment benefits. These distortions are well understood, so by zeroing them out we can focus attention directly on inefficiencies related to mismatch and OTJ search.

4.3.1 Mismatch and No OTJ Search

In this case, $0 < \eta_t^L < 1$ and $\pi = \eta_t^H = 0$. Allowing for permanent mismatch by shutting down OTJ search, the static labor wedge for low-tech employment reduces to:

$$\Omega_{Static,t}^L = \frac{\eta_t^L \frac{h_t^{L\prime}}{u_{c,t}^L} + \left(1-\eta_t^L\right)\frac{h_t^{M\prime}}{u_{c,t}^H}}{\eta_t^L \frac{h_t^{L\prime}}{u_{c,t}^L} + \left(1-\eta_t^L\right)\frac{h_t^{M\prime}}{u_{c,t}^H}\frac{\psi^L\left(1-\psi^H\right)}{\left(1-\psi^L\right)\psi^H}}.$$

The term in the denominator shows that any distortion in this static margin is driven by asymmetries in the parameterization of bargaining power across the two labor markets. Indeed, when $\psi^L \neq \psi^H$, the asymmetry across labor markets implies that permanent mismatch introduces a distortion into the static margin for low-tech employment even when the Hosios condition holds in both labor markets separately, so that $\xi^L = \psi^L$ and $\xi^H = \psi^H$. In contrast, as long as $\psi^L = \psi^H$ we have $\Omega_{Static,t}^L = 1$.

For high-tech employment, the static efficiency conditions in the private equilibrium coincide with those in the socially efficient equilibrium. Accordingly, the static labor wedge disappears so that $\Omega_{Static,t}^H = 1$. Similarly, shutting down OTJ search eliminates the wedge for high-tech job creation, so that $\Omega_{Dynamic,t}^H = 1$.

The dynamic wedge for low-tech employment reduces to:

$$\Omega_{Dynamic,t}^L = \frac{Y_{1,t} + \left(1-\eta_t^L\right)\Gamma_t^L + \left(1-\rho^L\right)\mathbb{E}_t\Xi_{t+1}\left[\frac{\gamma^L}{m_{v,t+1}^L} - \left(1-\eta_t^L\right)\Gamma_{t+1}^L - \frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L}\right]}{\begin{array}{c}\eta_t^L Y_{1,t} + \frac{1-\psi^H}{1-\xi^L}\left(1-\eta_t^L\right)Y_{2,t} \\ +(1-\rho^L)\mathbb{E}_t\left\{\Xi_{t+1|t}\left(\eta_t^L\left(\frac{1}{m_{s,t}^L}-1\right)\frac{h_t^{L\prime}}{u_{c,t}^L} + \left(1-\eta_t^L\right)\left(\frac{1}{m_{s,t}^L} - \frac{\psi^H}{\xi^L}\right)\frac{1-\psi^H}{1-\xi^L}\frac{\xi^L}{\psi^H}\frac{h_t^{M\prime}}{u_{c,t}^H}\right)\right\}\end{array}}.$$

With one unique exception (discussed below), as long as $0 < \eta_t^L < 1$ the low-tech job creation margin is distorted, so that $\Omega_{Dynamic,t}^L \neq 1$. This is true even when the Hosios condition holds both within and across markets, so that $\xi^L = \psi^L = \xi^H = \psi^H$.

Using the fact that equations (25) and (26) are linked, we can write the dynamic wedge for mismatch employment as:

$$\Omega^M_{Dynamic,t} = \frac{Y_{2,t} - \eta^L_t \Gamma^L_t + \left(1-\rho^L\right) \mathbb{E}_t \Xi_{t+1} \left[\frac{\gamma^L}{m^L_{v,t+1}} + \eta^L_{t+1}\Gamma^L_{t+1} - \frac{h^{L\prime}_{t+1}}{u^L_{c,t+1}}\right]}{Y_{1,t} + \left(1-\eta^L_t\right)\Gamma^L_t + \left(1-\rho^L\right)\mathbb{E}_t \Xi_{t+1}\left[\frac{\gamma^L}{m^L_{v,t+1}} - \left(1-\eta^L_{t+1}\right)\Gamma^L_{t+1} - \frac{h^{L\prime}_{t+1}}{u^L_{c,t+1}}\right]} \Omega^L_{Dynamic,t}.$$

Again, with one unique exception discussed immediately below, as long as $0 < \eta^L_t < 1$ the mismatch job creation margin is distorted, so that $\Omega^M_{Dynamic,t} \neq 1$.

Symmetry in the Market for Low-tech Jobs The one exception noted above is the special case of symmetry across the low- and high-tech job markets, so that $\psi^L = \psi^H = \xi^L = \xi^H$, and symmetry across the marginal productivities of low-skilled and mismatch labor, so that $Y_{1,t} = Y_{2,t}$. This implies $h^{L\prime}_{t+1}/u^L_{c,t+1} = h^{M\prime}_{t+1}/u^H_{c,t+1} = (m^L_{s,t}/m^L_{v,t})\gamma^L$, which, in turn, means $\Gamma^L_t = 0 \; \forall t$. Only in this one special case do we have $\Omega^L_{Static,t} = \Omega^H_{Static,t} = \Omega^L_{Dynamic,t} = \Omega^M_{Dynamic,t} = \Omega^M_{Dynamic,t} = 1$ with $0 < \eta_L < 1$. This special case will prove a useful benchmark in the numerical results presented later.

4.3.2 No Mismatch and No OTJ Search

In this case, $\eta^L_t = 1$ and $\pi = \eta^H_t = 0$. Shutting down mismatch entirely reduces the model to a two-sector model with completely segmented labor markets. Under our assumption of the Hosios condition and no unemployment benefits, the static wedges reduce to $\Omega^L_{Static,t} = \Omega^H_{Static,t} = 1$. This implies $h^{L\prime}_{t+1}/u^L_{c,t+1} = (m^L_{s,t}/m^L_{v,t})\gamma^L$ and $h^{H\prime}_{t+1}/u^H_{c,t+1} = (m^H_{s,t}/m^H_{v,t})\gamma^H$. We can substitute this in with the fact that $\eta^L_t = 1$ to show that the dynamic wedges collapse to $\Omega^L_{Dynamic,t} = \Omega^H_{Dynamic,t} = 1$ (of course, the mismatch dynamic wedge is irrelevant in this context). This result implies that in absence of mismatch the labor market efficiency conditions $i \in (H,L)$ reduce to:

$$h^i_{lfp,t}/u^i_{c,t} = \left(m^i_{s,t}/m^i_{v,t}\right)\gamma^i,$$

and

$$\gamma^i/m^i_{v,t} = Y_{i,t} + \mathbb{E}_t\Xi_{t+1}\left\{\left(1-\rho^i\right)\left[\left(\gamma^i/m^i_{v,t+1}\right)\left(1-m^i_{s,t+1}\right)\right]\right\}.$$

In other words, with the Hosios parameterization and zero unemployment benefits, shutting down both mismatch and OTJ search results in a private search equilibrium that is socially efficient. Indeed, both the static and dynamic efficiency conditions are identical to those presented in Arseneau and Chugh (2012) for the one sector general equilibrium labor search model. In this sense, our paper illustrates how the efficiency results presented in that earlier paper extend to a more general economy characterized by mismatch and OTJ search.

4.3.3 Congestion Externality and Unemployment Benefits

With both mismatch and OTJ search shut down (so $\eta_L = 0$ and $\pi = 0$), we reintroduce both the congestion externality, $\psi^i \neq \xi^i$, and unemployment benefits, $\chi^i > 0$, under the assumption of symmetry across labor markets, so that $\psi^L = \psi^H$. The static and dynamic distortions for $i \in (H,L)$ collapse to:

$$\Omega^i_{Static,t} = \frac{\psi^i}{1-\psi^i}\frac{1-\xi^i}{\xi^i}\frac{h^i_{lfp,t}/u^i_{c,t}}{h^i_{lfp,t}/u^i_{c,t} - \chi^i},$$

and

$$\Omega^i_{Dynamic,t} = \frac{Y_{i,t} + (1-\rho^i)\,\mathbb{E}_t\Xi_{t+1}\left\{\frac{\gamma^i}{m^i_{v,t+1}}\left(1 - m^i_{s,t+1}\right)\right\}}{\frac{1-\psi^i}{1-\xi^i}\left(Y_{i,t} - \chi^i\right) + (1-\rho^i)\,\mathbb{E}_t\Xi_{t+1|t}\left[\frac{\gamma^i}{m^i_{v,t+1}}\left(1 - \frac{\psi^i}{\xi^i}m^i_{s,t+1}\right)\right]}.$$

It is clear that either deviations from the Hosios condition ($\psi \neq \xi$) or positive unemployment benefits ($\chi > 0$) are sufficient to introduce a distortion to the competitive search equilibrium.

Taken together the preceding special cases demonstrate that the mismatch distortion is independent from the typical search-based distortions owing to congestion externalities and/or unemployment benefits. Moreover, allowing for OTJ search amplifies the mismatch distortion. That said, the complicated expressions for the static and dynamic distortions presented in Sections 4.1 and 4.2 clearly illustrate that all of these distortions interact in a complicated way in general equilibrium.

4.3.4 Frictionless Labor Markets

Lastly, it is useful to illustrate that in absence of search frictions the model collapses to a standard two-sector RBC model. To see this consider that we can shut down the long lived nature of employment relationships by making matches last only one period, so that $\rho^i = 1$. In this case, the dynamic efficiency conditions given by equations (20) and (22) reduce to a simple static relationship, $\gamma^i/m^i_{v,t} = Y_{i,t}$ for $i \in (H,L)$. Plugging this relationship into equations (18) and (19) gives $h^i_{lfp,t}/u^i_{c,t} = m^i_{s,t}Y_{i,t}$. Finally, in absence of search frictions effort expended by the household in the labor market is trivially translated one-for-one into new "matches" (though, to be clear, the concept of a labor market match is meaningless in absence of frictions), so that $m^i_{s,t} = 1$. Then, we retrieve the following expression: $h^i_t/u^i_{c,t} = Y_{i,t}$, which is the familiar efficiency condition at the heart of the (two sector) RBC model.

5 Quantitative Results

We calibrate the model to U.S. labor market data and conduct some simple experiments to gauge the size of the welfare effects of skill-mismatch.

5.1 Calibration

Our calibration, summarized in Table 1, is at weekly frequency. We use data on educational attainment to calibrate worker heterogeneity and data on employment by occupation to calibrate firm heterogeneity. We also make use of aggregate labor market data when applicable. All data are publicly available from the Bureau of Labor Statistics (BLS).

Table 1: Baseline parameterization (Weekly Frequency)

Preference Parameters	
Discount factor, β	0.999
Utility curvature, σ	1
Elasticity of participation, ε	1
Scaling for disutility of low-skill participation, ϕ^L	9.793
Scaling for disutility of mismatch participation, ϕ^M	86.570
Scaling for disutility of high-skill participation, ϕ^H	11.470
Production Parameters	
Aggregate technology, Z	0.396
Input-specific technologies, $z^M = z^L$	1
High-skill share in final goods, ϱ^H	0.507
Mismatch share in low-tech production, ϱ^L	0.550
Final goods input substitutability, ω_F	0.400
Low-tech input substitutability, ω_L	1
Labor Market Parameters	
Fraction of low-skill population, κ	0.500
Vacancy flow costs, $\gamma^H = \gamma^L$	0.133
Low-tech job destruction probability, ρ^L	0.012
High-tech job destruction probability, ρ^H	0.007
Low-tech matching efficiency, A^L	0.193
High-tech matching efficiency, A^H	0.164
Matching function elasticity, $\xi^L = \xi^H$	0.500
Worker bargaining power, $\psi^H = \psi^L$	0.500
Low-skill unemployment benefits, χ^L	0.045
High-skill unemployment benefits, χ^H	0.060
On-the-job search efficiency, π	0.136

The empirical counterparts to our low- and high-tech sectors are routine and nonroutine occupations, respectively, as per standard BLS occupational classifications. With this dichotomy in mind, we use the BLS occupational outlook handbook to obtain educational attainment requirements for entry-level positions by occupation. Roughly 82 percent of non-routine jobs require at least some post-secondary education, while only 14 percent of routine jobs require at least some post-secondary education. Accordingly, in our model high-skill workers are those with at least some post-secondary education and low-skill workers are those with at most a high school degree. Data from the BLS shows that about one-half the U.S. population has at most a high school degree, so we set the model economy's fraction of low-skill individuals $\kappa = 0.5$.

With regard to preferences, because we assume that the time period is equal to one week we set the discount factor $\beta = 0.999$, which is consistent with an annual interest rate of 5 percent. We assume a standard functional form for the sub-utility over consumption for both low- and high-skilled individuals:

$$u(c_t^i) = \frac{1}{1-\sigma}\left(c_t^i\right)^{1-\sigma} \text{ for } i \in (H, L).$$

and set $\sigma = 1$ for $i \in (H,L)$.

The sub-utilities over labor force activity for low- and high-skilled individuals, respectively are given by:

$$h(lfp_t^L) = \frac{\phi^L}{1+1/\varepsilon}\left(n_t^L + (1-f_t^L)s_t^L\right)^{1+1/\varepsilon},$$

and

$$h\left(lfp_t^H\right) + h\left(lfp_t^M\right) = \left[\begin{array}{c} \frac{\phi^H}{1+1/\varepsilon}\left(n_t^H + (1-f_t^H)s_t^H\right)^{1+1/\varepsilon} \\ +\frac{\phi^M}{1+1/\varepsilon}\left(n_t^M + (1-f_t^L)s_t^M\right)^{1+1/\varepsilon} \end{array}\right],$$

where: $\phi^i > 0$, for $i \in (H,L)$, and $\varepsilon > 1$ are parameters.

In calibrating preferences over labor force activity, quadratic labor disutility (so that $\varepsilon = 1$) implies that the model's aggregate labor force participation rate is highly inelastic with respect to output per worker, which is in line with the data.[15] BLS data show that the average participation rate of individuals with at least some post-secondary education is 1.33

[15] Our assessment of this elasticity comes from using quarterly data on real GDP from the Bureau of Economic Analysis and data on aggregate employment and the aggregate labor force participation rate from the BLS.

times as high as the participation rate of individuals with at most a high school education. Also, the average labor force participation rate in the U.S. is 0.631. We calibrate the scaling parameters, ϕ^L and ϕ^H, to target these participation-rate data. The scaling parameter for the disutility of mismatch employment for high-skilled individuals, ϕ^M, is calibrated to target a steady-state ratio of total employment in high-tech to low-tech jobs of $n^H/N^L = 1.11$. This number corresponds to the average ratio of total employment in nonroutine occupations to total employment in routine occupations in the U.S.

For production, we assume that output of final goods is a CES aggregate of the low- and high-tech intermediate good, so that:

$$Y_t = Z_t \left(\varrho^H \left(y_t^H \right)^{\omega_F} + \left(1 - \varrho^H \right) \left(y_t^L \right)^{\omega_F} \right)^{1/\omega_F},$$

where: Z_t is aggregate productivity; $\varrho^H \in (0,1)$ is the share of the high-tech intermediate input in final goods production; and ω_F governs the degree of substitutability between the high- and low-tech goods in final goods production. In turn, production of the high-tech good is given by $y_t^H = n_t^H$. Production of the low tech good is determined by the CES aggregator of low-skill and mismatch employment relationships:

$$y_t^L = \left(\varrho^L \left(z_t^L n_t^L \right)^{\omega_L} + \left(1 - \varrho^L \right) \left(z_t^M n_t^M \right)^{\omega_L} \right)^{1/\omega_L},$$

where: $\varrho^L \in (0,1)$ is the share of low-tech input; ω_L governs the substitutability of low and mismatch inputs; and z_t^L and z_t^M are input-specific technology parameters. The steady state values of z^L and z^M are normalized to 1. In contrast, the value of Z is chosen to normalize steady state aggregate output so that at quarterly frequency $Y = 1$. Also, we assume $\omega_L = 1$ so that low-skilled and mismatch workers are perfect substitutes in the production of the low-tech intermediate input. The remainder of the production parameters are either chosen based on the existing literature or calibrated to match empirically observed wage differentials.

For final output we follow Krusell, Ohanian, Rios-Rull, and Violante (2000) and set ω_F equal to 0.4. This value is also broadly in line with research surveyed in Hammermesh (1993). To calibrate the share parameter in the low-tech intermediate goods aggregator ϱ^L we set the equilibrium mismatch wage 15 percent above the low-skill wage based on Sicherman (1991). (We assume that a 4-year education differential is a reasonable characterization of the educational difference between high- and low-skill workers in the model economy.). For the share parameter in the final goods aggregator ϱ^H we draw on occupational wage data from the BLS. The employment-weighted median wages of individuals employed in nonroutine occupations is 1.35 times that of employment-weighted median wages of individuals employed in routine occupations. Accordingly, we choose ϱ^H so that $w^H/W^L = 1.35$.

Turning to the labor market, we assume that both the low- and high-tech job markets are characterized by a standard Cobb-Douglas matching function:

$$m_t^i = A^i \left(e_t^i\right)^{\xi^i} \left(v_t^i\right)^{1-\xi^i}, \text{ for } i \in \{L,H\},$$

where: A^i is matching efficiency; and ξ^i is the elasticity of the matching function with respect to total search search activity in a market, which we denote by e^i. We set $\xi^i = 0.5$ for $i \in \{L,H\}$, which is broadly in line with research surveyed in Petrongolo and Pissarides (2001).

The matching efficiency parameters, A^L and A^H, are jointly calibrated to hit empirical targets that we obtain from both aggregate and sector-specific data on job finding probabilities. Starting with the aggregate data and following the methodology in Elsby, Michaels, and Solon (2009) and Shimer (2012), monthly data on unemployment since 1951 reveal that the probability that an average unemployed individual matches with a job within a week is 0.132. Thus, one calibrating target for the two matching efficiency parameters is the steady-state value $\frac{(1-\eta^H)m^H + m^L}{s^L + s^M + s^H} = 0.132$. Moving to the sector-specific data, we find that since 2000 the average job-finding probability of individuals last employed in routine occupations is 0.99 times that of individuals last employed in nonroutine occupations. Assuming that an individual's last occupation is roughly indicative of their skill level, our second calibrating target for the matching efficiency parameters is the steady-state value: $\frac{m^L/(s^L + s^M)}{(1-\eta^H)m^H/s^H} = 0.99$.

The exogenous job destruction probabilities ρ^L and ρ^H are calibrated using BLS data on aggregate and occupation-specific unemployment rates. These data show that the average U.S. unemployment rate since 1951 is 0.058, so one of the job destruction rates is pinned down by targeting the steady-state ratio $(u^L + u^H)/(lfp^L + lfp^H) = 0.058$. In addition, these data also show that the average unemployment rate of individuals last employed in nonroutine occupations is about 1.62 times as high as that of individuals last employed in nonroutine occupations. So, we pin down the second job destruction rate by targeting the steady-state ratio $\frac{u^L}{lfp^L} / \frac{u^H}{lfp^H} = 1.62$.

We assume symmetry in the vacancy posting costs, $\gamma^H = \gamma^L$, and calibrate these costs to target the ratio of aggregate vacancies to aggregate unemployment: $\frac{v^L + v^H}{(1 - f^L)s^L + (1 + f^H)s^H} = 0.68$. The target for this ratio results from using data on aggregate job openings from the BLS Job Openings and Labor Turnover Survey since 2000 (when first available) combined with the Conference Board's Help-Wanted Index from 1951 through 2000 together with time series for aggregate U.S. unemployment.

In line with Shimer (2005), unemployment benefits are set to deliver a 40 percent replacement rate of wages. In particular, the low-skill unemployment benefit χ^L is pinned down by targeting the steady-state value $\chi^L = 0.4w^L$. Analogously, the high-skill unemployment benefit χ^H is pinned down by targeting the steady state value $\chi^H = 0.4\frac{w^H n^H + w^M n^M}{n^H + n^M}$. We also assume symmetry in bargaining power, so that $\psi^H = \psi^L = 0.5$. This parameterization has the virtue that, in our model, $\psi^H = \psi^L = \xi^H = \xi^L$ delivers both an efficient split of match surplus (see Hosios (1990)) as well as cross-market efficiency under permanent mismatch.

Finally, we calibrate the value for the on-the-job search efficiency parameter, π, following Nagypal (2005). Nagypal's findings suggest that for individuals with at least some post-secondary education (the empirical counterpart to our model's high-skill workers) the ratio of the average transition rate into new employment for those already employed to the average transition rate into new employment for those who are unemployed is in the order of 0.14. We select π to hit this target.[16]

5.2 Main Results

Table 2 presents the main results in the baseline economy for the private (Panel A) and socially efficient equilibrium (Panel B) at a quarterly frequency. One interesting result that arises endogenously from the calibration is that in the private economy $\pi^* = \pi > 0$

[16] In particular, within the context of our model Nagypal's equation (1) implies that for high skill workers:

$$\frac{\pi f^H \left(1 - u^H/lfp^H\right)}{\left((f^L s^M + f^H s^H)/(s^M + s^H)\right)\left(u^H/lfp^H\right)} = 2.82,$$

where the reference point 2.82 is the midpoint of 2.57 and 3.07, which is the range of values that Nagypal finds relevant for individuals with at least some post secondary education.

implying that the aggregate mismatch rate, n^M/N (the ratio of mismatch employment to total employment), is about 5 percent. This is very much in line with empirical results in Fallick and Fleischman (2004) who report a fraction of all employed individuals actively engaged in OTJ search equal to about 0.045. In contrast, while the planner chooses to allocate some high-skilled labor to mismatch employment in the efficient equilibrium, so that $n^M > 0$, as shown by the last row of the table under the baseline calibration the planner opts to shut down OTJ search entirely so that $\pi^* = 0$ and $\eta^H = 0$.

Table 2: Baseline Economy (Quarterly Frequency)

	A. Private Equilibrium		B. Efficient Equilibrium	
	Low	*High*	*Low*	*High*
Welfare Costs				
1.Hh. welfare	1.381	−0.183	–	–
2.Agg welfare	0.599		–	
Aggregate Variables				
3. c^L, c^H	0.480	0.480	0.477	0.477
4.LFP rate (L,H)	0.542	0.719	0.526	0.717
Labor Market Variables				
5. n^L, n^H	0.251	0.313	0.249	0.311
n^M	–	0.030	–	0.035
6. s^L, s^H	0.023	0.012	0.018	0.013
s^M	–	0.007	–	0.003
7. v^L, v^H	0.014	0.011	0.016	0.014
8. θ^L, θ^H	0.468	0.661	0.790	1.099
9. f^L, f^H	0.816	0.820	0.895	0.896
10. q^L, q^H	0.981	0.933	0.947	0.870
11. Unemp. rate (L,H)	0.075	0.046	0.056	0.035
12. η^L, η^H	0.772	0.249	0.877	0.00

Notes: Welfare costs (gains) are calculated as percent of steady state consumption required to give to (take away from) each household (low- and high-skilled, separately) in the private equilibrium to make them as well off as in the socially efficient equilibrium. Aggregate welfare costs are an equally weighted sum of the costs to low- and high-skilled households. Positive numbers indicate welfare costs and negative numbers indicate gains.

The household-specific welfare costs, measured as the percent of additional consumption that would be required to give to (or to take away from) each household to make them as well off in the private equilibrium as they are in the efficient equilibrium, are shown in line 1 of the table. The low-skilled household experiences significant welfare loss in the baseline calibration—nearly $1\frac{1}{2}$ percent of steady state consumption (positive numbers indicate welfare costs in the sense that a household must be compensated with additional consumption in order to be as well off in the private equilibrium as in the efficient equilibrium). In contrast, the high-skilled household experiences a modest welfare gain (negative numbers indicate welfare gains). Line 2 of the table shows the aggregate welfare cost, which is simply an equally-weighted average of the costs for the low- and high-skill households.

The remainder of the table presents the supporting set of allocations in each of the two equilibria. In the baseline calibration, firms post too few vacancies relative to the efficient outcome and both households devote too much search activity directed to the low-tech market. Accordingly, labor-market tightness in both the low- and high-tech sectors is suboptimally low (i.e., both θ^L and θ^H are too low relative to their respective efficient allocations). The corresponding implications for the job finding probability for workers and the job filling probability for firms are that the unemployment rate in the private equilibrium is suboptimally high and the level of employment is suboptimally low. Taking all of this into consideration, it is clear that the welfare costs to the low-skilled household stem from inefficiently high labor force participation. While the high-skilled household also incurs modest welfare costs from excessive labor force participation, these costs are more than offset by inefficiently high consumption, which is shared across households through complete financial markets.

5.2.1 Isolating the Welfare Effects of Mismatch

The results presented thus far are for the general case that includes unemployment benefits as well as mismatch with OTJ search; from the earlier analysis we know that all three are of these model features are distortionary. To isolate the welfare effects of skill-mismatch, in Table 3 we decompose the baseline welfare results into the component driven by each distortion in isolation.

Table 3: Incremental Effects of the Three Distortions (Quarterly Frequency)

	Percentage Point Owing to:							
	A. Perm. Mismatch		B. Trans. Mismatch		C. Unemp. Benefits		D. Total Percent	
	Welfare Costs							
1. Hh welf. (L, H)	0.132	-0.131	1.346	-0.417	-0.097	0.365	1.381	-0.183
2. Agg. welfare	0.001		0.464		0.134		0.599	
	Utility Subcomponents (difference between private and efficient equilibrium)							
3. c^L, c^H	> -0.001		-0.458		1.156		0.698	
4. LFP (level) (L,H)	0.193	-0.116	1.281	-0.792	1.528	1.269	3.003	0.361

Notes: Welfare costs (gains) are calculated as percent of steady state consumption required to give to (take away from) each household (low- and high-skilled, separately) in the private equilibrium to make them as well off as in the socially efficient equilibrium. Aggregate welfare costs are an equally weighted sum of the costs to low- and high-skilled households. Positive numbers indicate welfare costs and negative numbers indicate gains. On a row-wise basis Panels A through C sum up to Panel D.

The top half of Panel D in the table reproduces results already reported in Table 2. Under the baseline calibration: the low-skill household experiences welfare costs in the order of $1\frac{1}{2}$ percent of consumption; the high skill household experiences modest welfare gains; and the aggregate welfare costs are somewhat over $\frac{1}{2}$ percent of consumption. In turn, the bottom half of Panel D in Table 3 focuses on differences in key variables between the private and efficient solutions, showing that in the baseline calibration: consumption is nearly $\frac{3}{4}$ percent above its efficient level; low-skill labor force participation is about 3 percent above its efficient level; and high-skill labor force participation is a bit less than $\frac{1}{2}$ percent below its efficient level.

The remainder of Table 3 parses out the baseline welfare results by holding all other parameters in the model constant and showing the incremental distortionary effects caused by permanent and temporary mismatch and, separately, the unemployment benefit. Adding the incremental effects across Panels A though C explains the total effects reported in Panel D.

The distortionary effects of permanent mismatch are isolated in Panel A of the table by shutting down both the unemployment benefit and OTJ search in the private economy. The upper half of the table shows that for high-skilled households permanent mismatch creates welfare gains in the order of one-tenth of a percentage point of steady state consumption; these welfare gains are purely distributional and come at the expense of low-skilled households. The distributional impact stems from the fact that efficiency requires some substitution out of low-skilled labor and into mismatched labor in the low-tech sector. The lower half of the table reveals that in the private equilibrium this substitution in not aggressive enough. Low-skilled participation is nearly 20 basis points too high relative to the efficient equilibrium and high-skilled participation—largely owing to lack of activity in the market for mismatch jobs—is over 10 basis points too low. Given that the impact on consumption, which is shared across households through the complete financial markets, is minimal, the resulting welfare costs/gains are purely distributional with high-skilled households gaining at the expense of low-skilled households.

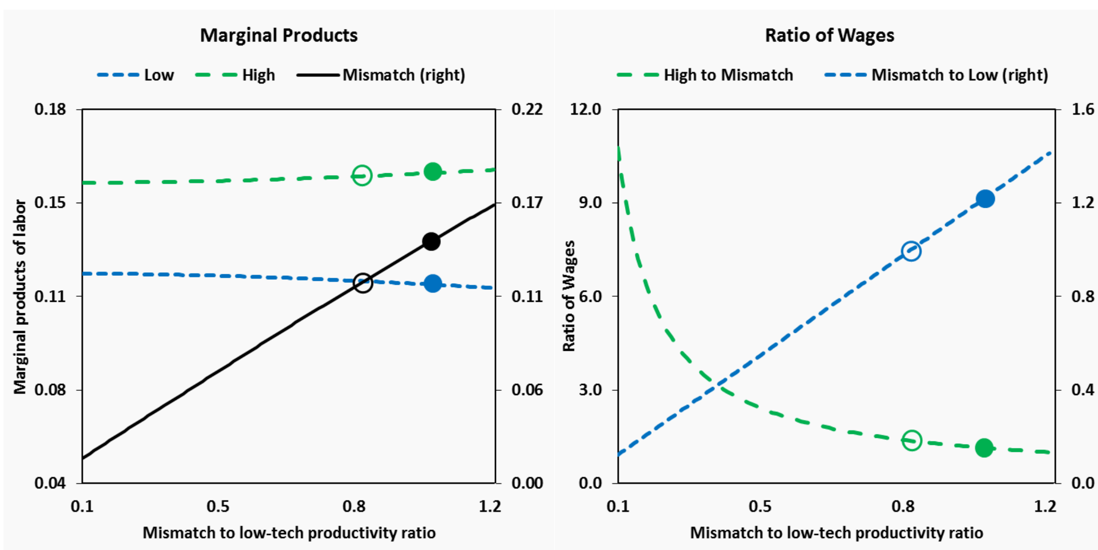
Panel B isolates the distortion associated with the temporary nature of mismatch by re-introducing OTJ search in the private economy. OTJ search increases the flow out of mismatch employment. In doing so, it lowers the cost of mismatch employment to the high-skilled household by shortening the expected length of time that a worker spends in mismatch employment (and therefore accepting a lower wage) before potentially moving to a higher paying job in the high-tech sector. As a result, the high-skilled household finds it optimal to devote a greater amount of search activity toward the market for mismatch jobs. This increase in search activity for mismatch jobs notwithstanding, sharp outflows through job-to-job transitions from successful OTJ search cause an overall decline in the stock of mismatch jobs. It is this later channel that amplifies the welfare effects of permanent mismatch as the presence of OTJ search further dampens the substitution out of low-skilled labor beyond what is efficient. Ultimately, this substitution causes participation by high-skilled households to be driven farther below its socially optimal level resulting in welfare gains in the order of $\frac{1}{2}$ percent of steady state consumption. On the other hand, low-skilled agents must work more in order to countervail the loss in production that, all else equal, stems from decline a in the number of mismatch jobs. The welfare costs to low-skilled individuals owing to the temporary nature of mismatch exceed $1\frac{1}{4}$ percent of steady state consumption.

Finally, Panel C shows the incremental welfare costs when unemployment benefits are added back to the private economy, taking us back to the baseline economy which is shown in Panel D. Broadly speaking, introducing unemployment benefits does two things from the point of view of workers, regardless of whether they are low- or high-skilled. First, it increases the incentive to search because the opportunity cost of leisure is higher. Second, it raises the outside option for workers in wage negotiations. In turn, this pushes up wages and reduces the willingness of firms to hire. Taken together, the increase in search activity (unemployment) outweighs the reduction in hiring (employment) leading to higher participation for both households. From a welfare perspective, both households benefit from higher consumption, which is supported directly by the unemployment benefit and indirectly through reduced resources devoted to posting vacancies, but suffer from a reduction in leisure. For low-skilled households, the baseline parameterization is such that the former marginally dominates the later, generating modest welfare gains on the order of 20 basis points. In contrast, the opposite is true for high-skilled households, where the resulting welfare costs are nearly $\frac{1}{2}$ percent of steady state consumption.

5.3 A Closer Look at the Distributional Effect

In order to shed more light on the distributional effect of mismatch, we analyze how an increase in mismatch participation spills over to low-skilled individuals. In line with panel A of Table 3 we assume permanent mismatch, $\pi = \eta_t^H = 0$, no unemployment benefits, $\chi = 0$, and Hosios efficiency with $\psi^L = \xi^L = \psi^H = \xi^H$. We conduct a simple experiment in which the incentives for mismatch job formation are increased by raising the relative productivity of mismatched workers in the production of the low-tech intermediate good. Specifically, holding all other parameters in the model constant at the baseline calibration, we increase the mismatch productivity parameter, z^M, from a very low level to an upper bound determined by the point at which the mismatch wage converges to the wage earned in high-tech employment. The resulting range for relative productivity of mismatch to low-skilled individuals is $z^M/z^L \in [0.1, 1.15]$.

Figure 1: Marginal products and wage ratios



Notes: In each panel a solid dot denotes private outcomes under the benchmark calibration for z^M/z^L. A hollow dot denotes outcomes under the special case of efficiency in which the private and planning equilibria coincide with equalized mismatch and low-skill marginal products. In all cases the Hosios condition holds within and across markets, OTJ search-efficiency π is equal to zero, and unemployment benefits are equal to zero.

The left panel of Figure 1 shows how the shift in relative productivity drives the marginal product of low-skilled, mismatch, and high-skilled labor in the production of the final good in the private equilibrium. At the baseline calibration (denoted by the solid dots) $z^M/z^L = 1$.

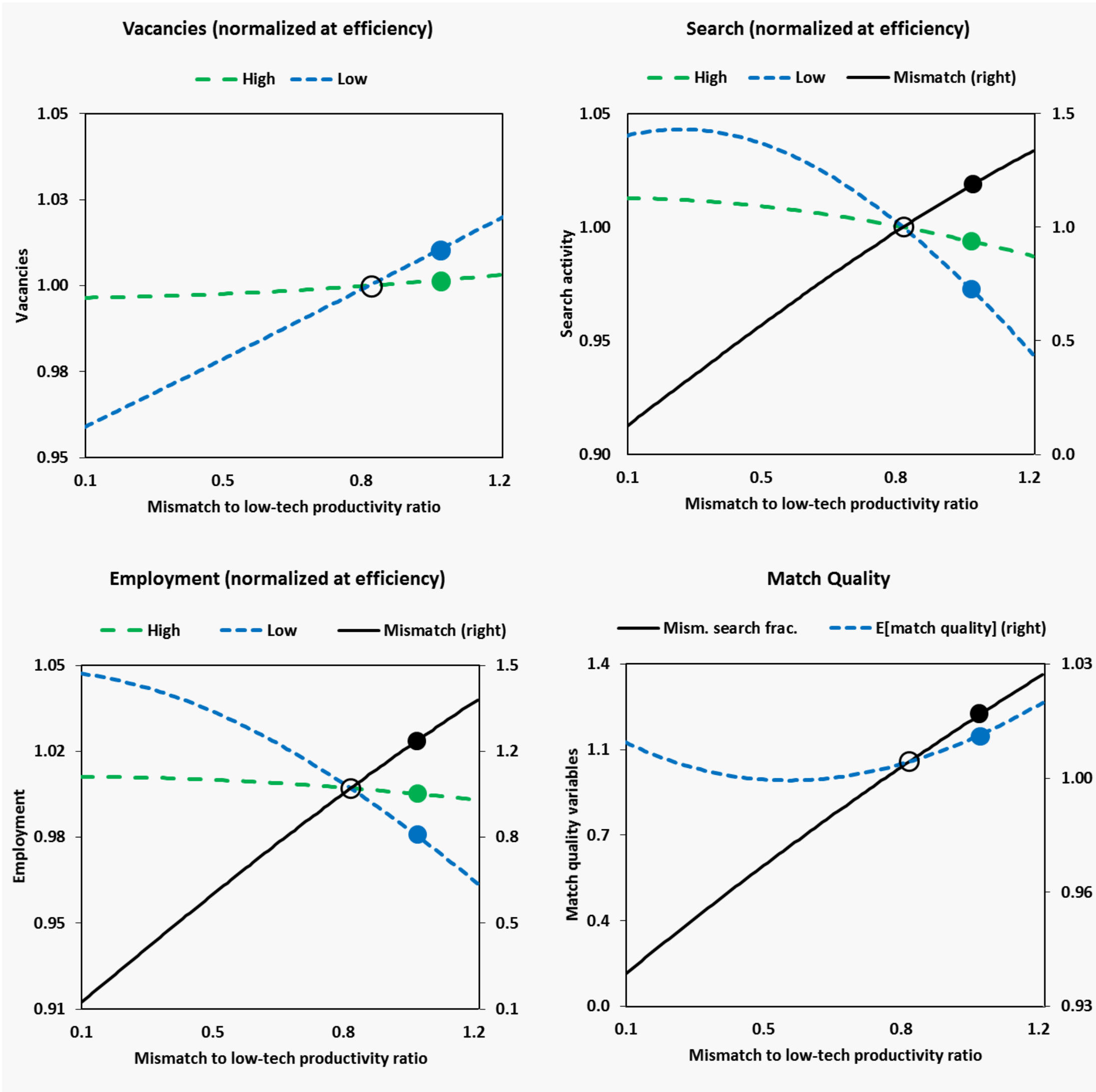
Given the share parameter ϱ^L at the baseline calibration (chosen so that $w^M/w^L = 1.15$, which is shown by the corresponding solid dot in the right panel of Figure 1) and the equilibrium values of n^L and n^M, the resulting ratio of the marginal products of mismatch and low-skilled labor is $Y_2/Y_1 = 1.22$. The hollow dots denote the point at which the marginal products are equal, so that $Y_2/Y_1 = 1$. This point, where $z^M/z^L < 1$ (in the left panel) and $w^M/w^L = 1$ (right panel), is a useful benchmark because it is the special case of efficiency with symmetry in the market for low-tech jobs that was examined in Section 4.3.1.

Finally, note from the right panel of Figure 1 that as mismatch productivity gets very high so that z^M/z^L approaches its upper bound, the high-tech wage premium over mismatch employment, w^H/w^M, approaches one from above. We rule out the part of the parameter space where $w^H/w^M < 1$ as empirically implausible. Similarly, as z^M/z^L approaches its lower bound, the mismatch wage premium over low-skilled employment approaches zero and the high-skilled wage premium over mismatch employment asymptotes to infinity. Our model is one of vertical mismatch, that is, one in which mismatch is a situation where a worker is overqualified for their job.[17] As discussed in the calibration section, empirical results from Sicherman (1991) suggest that overqualified individuals obtain a wage premium over low-skilled workers when engaged in mismatch employment. This suggests that any parameterization with $z^M/z^L < 1$, which implies $w^M/w^L < 1$, is inconsistent with our model of vertical mismatch.[18]

[17] Vertical mismatch refers to a situation in which, for example, a highly educated worker is employed in a job that requires only a high school degree to perform. In such a situation, an absolute advantage of the highly educated worker implies that he or she simply preforms the task better than an employee with less education and earns a higher wage as a result.

[18] In contrast, horizontal mismatch refers to mismatch within skill groups—for example, a low-skilled worker may have a comparative advantage in one low-skill task relative to another. If that worker is employed in a job that is not a source of comparative advantage, then we would expect the wage in that mismatch job to be lower. Indeed, McLaughlin and Bils (2001) find that workers who, upon switching jobs, also switch industries earn an employment-by-industry weighted average of roughly 15 percent less than job stayers, which is consistent with a wage differential of $w^M/w^L = 0.85$. This result suggests that the wage differential for horizontal mismatch reflects comparative advantage. Epstein (2012) studies horizontal mismatch further.

Figure 2: Match quality, search activity, vacancies, and employment



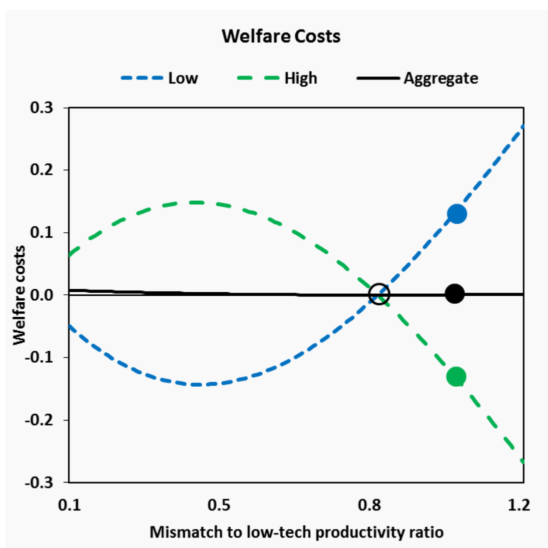
Notes: In each panel a solid dot denotes private outcomes under the benchmark calibration for z^M/z^L. A hollow dot denotes outcomes under the special case of efficiency in which the private and planning equilibria coincide with equalized mismatch and low-skill marginal products. In all cases the Hosios condition holds within and across markets, OTJ search-efficiency π is equal to zero, and unemployment benefits are equal to zero.

Figure 2 shows key labor market aggregates. The top left panel shows that as relative mismatch productivity increases both low- and high-tech firms increase vacancy postings, with the former rising more rapidly than the later. In contrast, the top right panel shows that search activity responds differently in low- versus high-skilled households. High-skilled households devote more search activity to the market for low-tech jobs while low-skilled households devote less effort. This differential household response results in a substitution away from low-skilled and into mismatched labor as shown in the bottom left panel. Indeed, as mismatch productivity rises mismatch search activity becomes a larger fraction of total search in the market for low-tech jobs (that is, $1 - \eta_t^L$ increases, as shown by the solid line in the bottom right panel) and match quality increases (the dashed line in the bottom right panel). All told, as the productivity of mismatch labor increases there is a substitution out of low-skilled and into mismatch labor. But, is this substitution efficient? We answer this question immediately below.

5.3.1 *Low-skilled households endure welfare costs...*

We now solve for the efficient allocations at each value of relative productivity for $z^M/z^L \in [0.1, 1.15]$ and calculate the associated welfare costs (gains) for both low- and high-skilled households (we continue to assume, as in Panel A of Table 3, permanent mismatch, $\pi = \eta_t^H = 0$, no unemployment benefits, $\chi = 0$, and Hosios efficiency with symmetric search markets, $\psi^L = \xi^L = \psi^H = \xi^H$).

Figure 3: Welfare costs



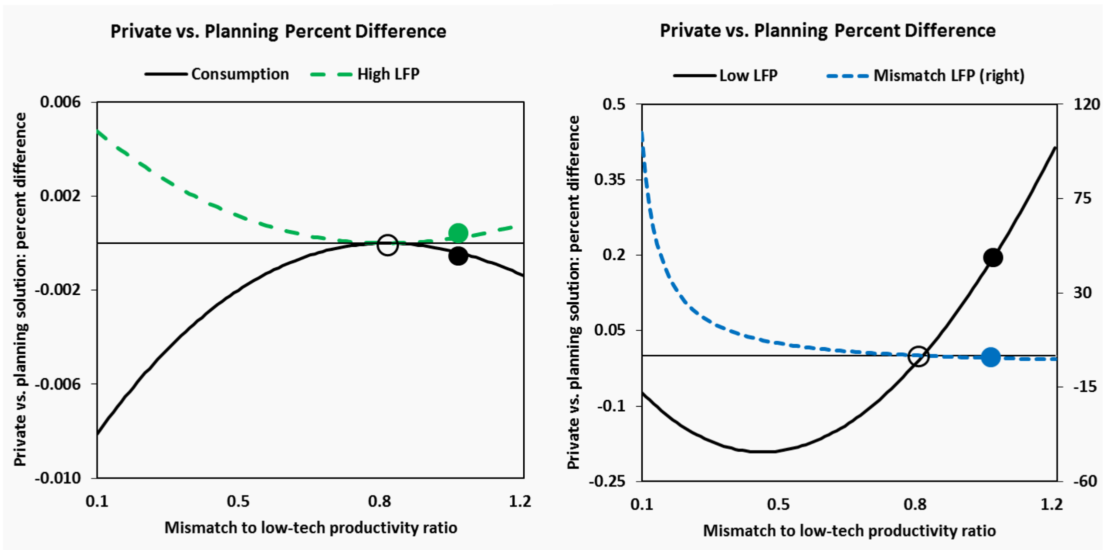
Notes: In each panel a solid dot denotes private outcomes under the benchmark calibration for z^M/z^L. A hollow dot denotes outcomes under the special case of efficiency in which the private and planning equilibria coincide with equalized mismatch and low-skill marginal products. In all cases the Hosios condition holds within and across markets, OTJ search-efficiency π is equal to zero, and unemployment benefits are equal to zero. Welfare costs (gains) are calculated as percent of steady state consumption required to give to (take away from) each household (low- and high-skilled, separately) in the private equilibrium to make them as well off as in the socially efficient equilibrium. Aggregate welfare costs are an equally weighted sum of the costs to low- and high-skilled households. Positive numbers indicate welfare costs and negative numbers indicate gains.

The results are plotted in Figure 3. The solid dots denote the welfare effects reported in Panel A of Table 3 at the baseline parameterization $z^M/z^L = 1$. Again, the hollow dot is the point at which $Y_2/Y_1 = 1$ All three lines cross zero at this point indicating efficiency in the special case of symmetry in the market for low-tech jobs (as established in Section 4.3.1).

The lines for low- and high-skilled households are mirror images of one another reflecting the purely distributional welfare effects of mismatch with one household gaining at the expense of the other. Additionally, note that whether the low-skilled household is gains or loses as a result of mismatch depends importantly on the relative productivity of high-skilled workers in mismatch jobs. Specifically, when $z^M/z^L > 1$ (as in the baseline calibration) mismatch benefits high-skilled households at the expense of the low-skilled household, whereas the opposite is true when $z^M/z^L < 1$. Again, we stress that the empirically relevant part of the parameter space for our model of vertical mismatch in one in which $z^M/z^L > 1$.

Figure 4 delves deeper into the role of relative productivity. Both panels show the percent difference between the private and socially efficient equilibrium for all components of the utility function at each value of z^M/z^L. This difference is zero at the hollow dot in both panels indicating an efficient private equilibrium in this special case in which $Y_2/Y_1 = 1$. Beginning with consumption (the solid line in the left panel), note that as long as z^M/z^L differs from 1—implying that w^M/w^L also differs from 1—consumption is always suboptimally low. However, this cost is shared across households through complete financial markets and thus cannot explain the distributional welfare results. Instead, the right panel reveals that low-skilled participation is suboptimally high when $z^M/z^L > 1$ and high-skilled participation in the mismatch market is suboptimally low. In other words, the distributional effect at the baseline parameterization of z^M/z^L stems from the fact that the substitution toward mismatch labor in the private equilibrium is not aggressive enough relative to the efficient equilibrium (in the case of high-skilled households, note that the welfare gains from inefficiently low mismatch participation (right panel of Figure 4) outweigh the costs from inefficiently high participation in the high-tech market (left panel of Figure 4)). While it is possible for the sign of the distributional effect to switch—so that low-skilled individuals gain at the expense of the high-skilled—such an outcome requires $z^M/z^L < 1$. We have already argued that this is at odds with our model of vertical mismatch because it requires a wage discount for mismatched workers relative to low-skilled workers.

Figure 4: Percent differences between private and planning solutions



Notes: In each panel a solid dot denotes private outcomes under the benchmark calibration for z^M/z^L. A hollow dot denotes outcomes under the special case of efficiency in which the private and planning equilibria coincide with equalized mismatch and low-skill marginal products. In all cases the Hosios condition holds within and across markets, OTJ search-efficiency π is equal to zero, and unemployment benefits are equal to zero.

5.3.2 *...and incremental mismatch crowds out welfare.*

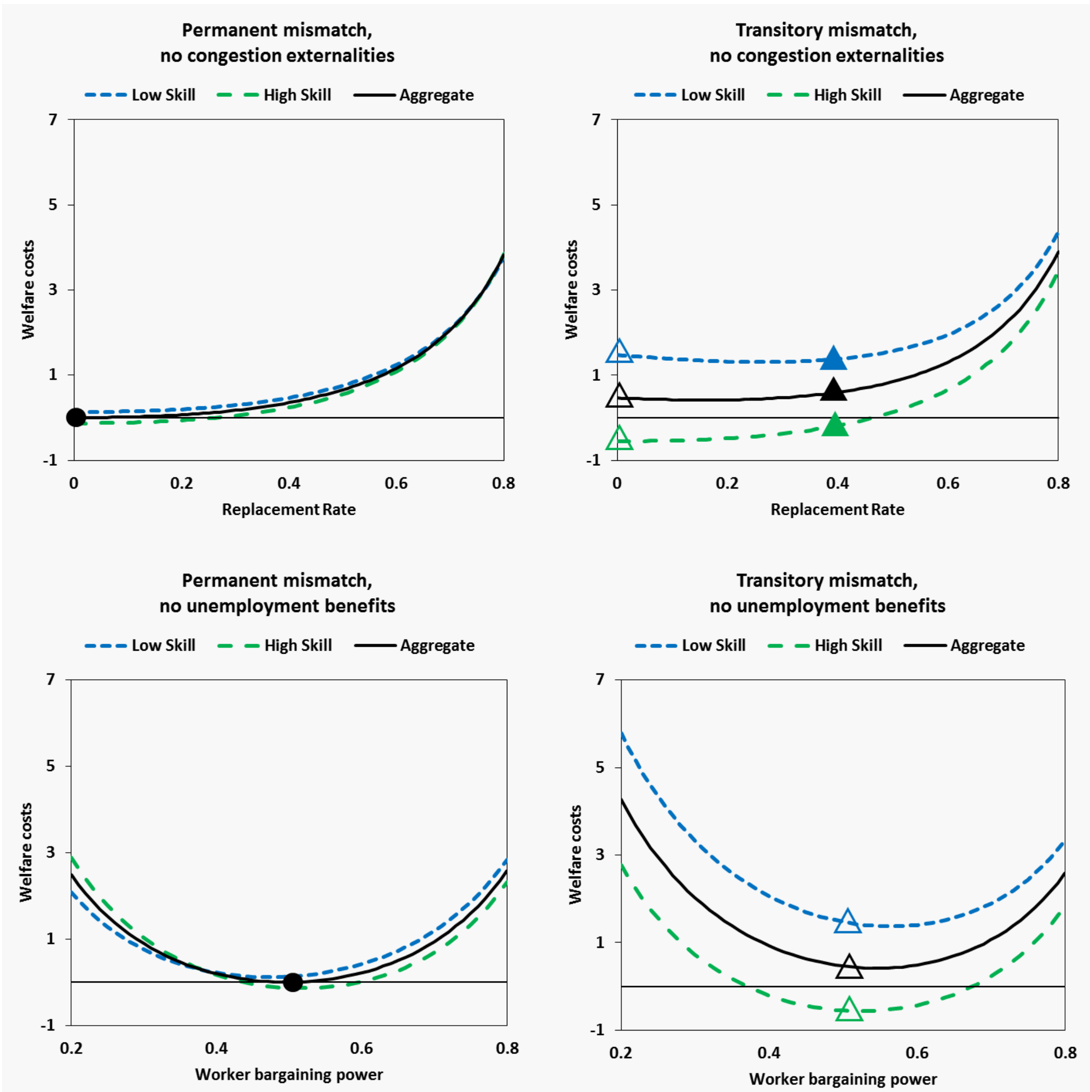
A final point to take away from this experiment has to do with the effect of incremental mismatch activity on low-skilled welfare. Figure 3 shows that for sufficiently high levels of z^M/z^L incremental mismatch activity diminishes the welfare of low-skilled households (the slope of the line indicating welfare effects for low-skill households is positive, regardless of whether it lies above (indicating welfare costs) or below (indicating welfare gains) zero). Thus, additional search activity by high-skilled individuals in the low-tech job market crowds out low-skilled labor and in doing so it also crowds out low-skilled welfare.

That said, we note that it is theoretically possible to get "crowding in" of low-skilled welfare. Indeed, over a sufficiently low level of z^M/z^L, in Figure 3 the slope of the line indicating welfare effects for low-skill households—which lies below zero over this range, indicating welfare gains—is negative. However, such an outcome requires an implausibly low level of w^M/w^L (see the right panel of Figure 1).

5.4 The Relative Size of the Mismatch Distortion

Our theoretical results show a complicated interaction between the distortions generated by mismatch, the unemployment benefit, and the size of the congestion externality. Figure 5 explores this interaction in greater detail.

Figure 5: Interaction with congestion externality and unemployment benefits



Notes: In each panel a solid dot denotes private outcomes under the benchmark calibration with zero unemployment benefits OTJ search efficiency $\pi = 0$ (Panel A of Table 3). A hollow triangle denotes the same economy with $\pi > 0$ (Panel B of Table 3). A solid triangle denotes the full baseline economy presented in Panel A of Table 2, which is equivalent to Panel D of Table 3. Welfare costs (gains) are calculated as percent of steady state consumption required to give to (take away from) each household (low- and high-skilled, separately) in the private equilibrium to make them as well off as in the socially efficient equilibrium. Aggregate welfare costs are an equally weighted sum of the costs to low- and high-skilled households. Positive numbers indicate welfare costs and negative numbers indicate gains.

The top two panels of Figure 5 present welfare calculations for different values of the replacement rate (the baseline assumption is $\chi = 0.4$) assuming the Hosios condition holds within and across markets ($\psi^i = \xi^i \; \forall i \in \{L, H\}$) in order to isolate the interaction of the mismatch distortion with the distortion generated by unemployment benefits. The hollow triangles in both the upper and lower right panels indicate the same equilibrium shown in Panel B of Table 3. The solid triangles shown in the top right panel indicate the baseline equilibrium described in Panel A of Table 2 and Panel D of Table 3. As before, the solid dot in the left panels denotes the private outcomes under the benchmark calibration with zero unemployment benefits and OTJ search efficiency $\pi = 0$.

The top left panel of Figure 5 considers the case in which mismatch is permanent. The plot shows that there are small distributional effects associated with permanent mismatch in absence of the unemployment benefit (the spread between the dotted and dashed line indicates that high-skilled households gain at the expense of low-skilled households). But, these distributional effects diminish as the replacement rate rises (the spread collapses as we move farther along the x-axis). Indeed, for a sufficiently high replacement rate the distributional effects are overwhelmed by the welfare costs associated with the unemployment benefit. The top right panel conducts the same exercise for the case in which mismatch is transitory. The amplification of the welfare costs of transitory mismatch stands out, particularly for low replacement rates. The distributional effects—the spread between the dotted and dashed lines—are more pronounced and the overall costs to low-skilled individuals are larger. That said, qualitatively the story is similar in that the distortion created by the unemployment benefit eventually dominates as the replacement rate rises regardless of whether mismatch is permanent or transitory.

The lower two panels of Figure 5 conduct a similar exercise to shed light on the interaction of the mismatch distortion with the congestion externality by varying worker's bargaining power, ψ^i (the baseline assumption is $\psi^i = \xi^i = 0.5 \; \forall i \in \{L, H\}$), under the assumption that $\chi^i = 0$. For permanent mismatch, the bottom left panel shows the welfare effects are minimized for either household in the neighborhood of our baseline parameterization.[19] As

[19]Note that the solid dot in both the upper and lower left panels indicate the same equilibrium shown in Panel A of Table 3.

bargaining power moves farther away from the Hosios condition, the distributional effects get larger but they are ultimately small relative to the total costs of the growing congestion externality. Finally, the bottom right panel shows that the amplification owing to transitory mismatch preserves the distributional aspect of the welfare costs over the range $0.35 < \psi^i < 0.65$. But, outside that range the congestion externality dominates so that as the bargaining share moves farther from the Hosios parameterization, both households suffer welfare costs.

6 Conclusion

This paper analyzes the welfare costs of mismatch employment. It makes two main contributions. First, we derive a set of efficiency conditions that provide a complete characterization of the distortions generated by skill-mismatch. Second, we measure the size of these distortions in a carefully calibrated version of the model that matches a number of aspects of the occupational and skill-based heterogeneity found in U.S. labor markets. We find the welfare effects of skill-mismatch to be distributional and quantitatively large. In addition, over the empirically plausible parameter space higher skill-mismatch generates welfare gains for high-skilled households, but these gains come at the expense of low-skilled households who experience a crowding out of labor activity.

There are a number of possible extensions. First, it would be interesting to introduce incomplete financial markets to move away from consumption risk sharing across low- and high-skilled households. It would also be interesting to extend the model to allow for lock-in to mismatch employment due to skill deterioration in the spirit of Pissarides (1994). Finally, although we have defined and measured the distortions associated with mismatch, we have not examined the design of optimal labor market policy to address these distortions. We leave these extensions for future research.

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